Instructions: (100 points) Solve each problem and box in your final answer.

Some Formulas: Below are several formulas, listed here for your benefit.

- \( P(A \text{ and } B) = P(A) \cdot P(B | A) \)
- \( \sigma^2 = \sum x^2 P(x) - \mu^2 \)
- \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \)

1. Short Responses. Respond to each of the questions below: multiple choice, and True (T) and False (F).

(a) A local golf store sells a bag containing 80 used golf balls (for practicing). Suppose the bag contains 35 Titleists, 25 Maxflis, and 20 Top-Flights. If a ball is selected at random from the bag, what is the probability the ball is not a Titleists ball?

- A 0.3125
- B 0.4375
- C 0.5625
- D 0.6875
- E None of these

(b) If one card is drawn from a standard deck of 52 playing cards, what is the probability the card is red three (3)?

- A \( \frac{2}{52} \)
- B \( \frac{1}{2} \)
- C \( \frac{2}{13} \)
- D \( \frac{4}{52} \)
- E None of these

(c) T (T or F) If two events, A and B, are mutually exclusive, then \( P(A \text{ or } B) = P(A) + P(B) \).

(d) Let A and B be two events such that \( P(A | B) = P(A) \). Then these events are...

- A mutually exclusive
- B not mutually exclusive
- C dependent
- D independent
- E not enough information

(e) From a group of 15 people, The method to calculate the number of ways we can choose a president, vice-president, and treasurer from the 15 people is

- A \( 3P_{15} \)
- B \( 3C_{15} \)
- C \( 15P_3 \)
- D \( 15C_3 \)
- E 3!

2. Simple Calculations. Use the appropriate counting formula to answer each of the questions below. Use the notation \( n! \), \( nP_r \), and \( nC_r \) to label your answer. (For example, an answer might be written as \( 3! = 6 \).

(a) A man has eight shirts, five pants, six pairs of sox, two belts, and three pairs of shoes. Assuming that they all match, how many different ways can he dress using his various shirts, pants, sox, belts, and shoes?

The number of ways is \( 8 \cdot 5 \cdot 6 \cdot 2 \cdot 3 = 1440 \)

(b) The teacher of a sociology class of 24 students wants to select 12 class members for a sociology project due at midterm. How many ways can she select 12 of her students to do the project?

The number of ways is \( 24C_{12} = 2,704,156 \)

(c) Suppose 20 divers compete in a high-diving competition. How many ways can 3 divers be chosen by the judges to receive the gold, silver, and bronze metals?

The number of ways is \( 20P_3 = 6,840 \)
3. Determine the probability distribution’s missing probability value.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x)$</td>
<td>0.07</td>
<td>0.20</td>
<td>0.38</td>
<td>0.22</td>
<td>0.13</td>
</tr>
</tbody>
</table>

4. An experiment consists of spinning two spinners: one with three colors red (R), green (G), and blue (B); the other with four colors red (R), green (G), blue (B), and yellow (Y). An outcome is a sequence of two letters, for example, RG indicates the first spinner stopped on red, and the second spinner stopped on green.

(a) (4 pts) List the sample space for this experiment

$S = \{ \text{RR, RG, RB, RY, GR, GG, GB, GY, BR, BG, BB, BY} \}$

(b) (3 pts) Compute the probability of not getting a yellow color on either spinner.

$P(\text{not a yellow}) = \frac{9}{12} = 0.75$

(c) (3 pts) Compute the probability of obtaining different colors on the two spinners.

$P(\text{different colors}) = \frac{9}{12} = 0.75$

(d) (2 pts) Compute the probability of obtaining at least one yellow or at least one green.

$P(\text{at least one yellow or at least one green}) = \frac{8}{12} = \frac{2}{3} \approx 0.667$

5. In a box of 48 light bulbs, 6 of the bulbs are defective. Two bulbs are selected – one at a time – at random without replacement. Use the Multiplication Rule to find the probability the first bulb chosen is good and the second one is defective.

$P(\text{good bulb on first and defective on second}) = \frac{42}{48}(6/47) \approx 0.1117$

6. Consider the probability distribution of a random variable $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x)$</td>
<td>0.15</td>
<td>0.20</td>
<td>0.15</td>
<td>0.25</td>
<td>0.25</td>
<td>1.00</td>
</tr>
<tr>
<td>$x \cdot P(x)$</td>
<td>0.15</td>
<td>0.40</td>
<td>0.45</td>
<td>1.00</td>
<td>1.25</td>
<td>3.25</td>
</tr>
</tbody>
</table>

(a) (7 pts) Fill in the rest of the entries (in row 3) in the table, and use the entries to compute the mean of the random variable $x$.

$\mu = 3.25$

(b) (3 pts) $P(2 < x \leq 5) = 0.65$

Solution: Calculated using $P(3) + P(4) + P(5)$
7. A fair die is rolled and a card is selected at random from a standard playing deck. Find the probability of rolling an even number and drawing a face card (Jack, Queen, or King).

\[ P(\text{roll an even number and a face card}) = \frac{1}{2} \times \frac{12}{52} = \frac{6}{52} = 0.1154 \]

8. The table shows the results of a survey that asked 2850 people whether they were involved in any type of charity work. A person is selected at random from the sample. Find the probability of each event.

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequently</td>
<td>231</td>
<td>217</td>
<td>448</td>
</tr>
<tr>
<td>Occasionally</td>
<td>466</td>
<td>440</td>
<td>906</td>
</tr>
<tr>
<td>Not at all</td>
<td>785</td>
<td>751</td>
<td>1536</td>
</tr>
<tr>
<td>Total</td>
<td>1482</td>
<td>1408</td>
<td>2890</td>
</tr>
</tbody>
</table>

In the questions below, we abbreviate the events, for example, the event of choosing a person that is frequently or occasionally involved in charity work is abbreviated as “frequently or occasionally.”

(a) \( P(\text{frequently or occasionally}) = \frac{1354}{2890} = 0.4685 \)

(b) \( P(\text{frequently} \mid \text{male}) = \frac{231}{1482} \approx 0.1559 \)

(c) \( P(\text{female} \mid \text{not at all}) = \frac{751}{1536} \approx 0.4889 \)

(d) \( P(\text{occasionally or male}) = \frac{1922}{2890} \approx 0.6651 \)

9. A statistics conference has an attendance of 6523 people. Of these, 4623 are college professors and 2632 are female. Of the college professors, 1045 are female.

\[ P(\text{female or college professor}) = 0.9520 \]

Work area.

\[ P(F \text{ or } C) = P(F) + P(C) - P(F \text{ and } C) = \frac{2632}{6523} + \frac{4623}{6523} - \frac{1045}{6523} = \frac{6210}{6523} \approx 0.9520 \]
(4pts) 10. A doctor gives a patient a 68% chance of surviving bypass surgery after a heart attack. If the patient survives the surgery, he has a 55% chance that the heart damage will heal. Find the probability that the patient survives surgery and the heart heals.

\[ P(\text{survives and heart heals}) = (.68)(.55) = 0.374 \]

Work area. We apply the multiplication rule,

\[ P(\text{survives and heart heals}) = P(\text{survives}) \cdot P(\text{heart heals | survives}) = (.68)(.55) = 0.374 \]

(5pts) 11. An urn contains 20 balls, 12 red and 8 blue. Five balls are drawn at random without replacement. Compute, using the combinatorial formulas, the probability that three red and two blue balls are drawn.

\[ P(\text{three red and two blue}) = 0.3973 \]

Work area. We have

\[ P(\text{three red and two blue}) = \frac{12C_3 \cdot 8C_2}{20C_5} = \frac{220 \cdot 28}{15504} = \frac{6160}{15504} \approx 0.3973 \]

Problems 12–13 concern the binomial distribution.

(6pts) 12. Let \( x \) be a binomial random variable on 75 trials with probability of success 0.58. Compute the mean and variance of \( x \).

\[ \mu = 43.5 \quad \text{Recall that } \mu = np = (75)(.58) = 43.5 \]

\[ \sigma^2 = 18.27 \quad \text{Recall that } \sigma^2 = npq = (75)(.58)(.42) = 18.27 \]

(11pts) 13. Fifty-two percent of businesses in the U.S. require a doctor's note when an employee takes sick time. You randomly select 16 businesses and ask each if it requires a doctor's note when an employee takes sick. Let \( x \) be the number of businesses (of the 16) that requires a doctor's note.

(a) (2 pts) Model the random variable, \( x \), as a binomial r.v. State the values of \( n \) and \( p \).

\[ n = 16 \quad p = 0.52 \]

(b) (3 pts) Calculate \( P(x = 8) = 0.1939 \)

(c) (3 pts) Calculate \( P(x \leq 8) = 0.5343 \)

(d) (3 pts) Calculate \( P(x \text{ is at least 8} = 0.6595 \)

Work area. We have

\[ P(x \text{ is at least 7}) = P(x \geq 8) = 1 - P(x < 8) = 1 - P(x \leq 7) = 1 - \text{binomcdf}(16,.52,7) = 1 - 0.3405 = 0.6595 \]