And Statements

- Symbol: \( \wedge \)
- Key word: AND
- Known as a conjunction

Or Statements

- Symbol: \( \vee \)
- Key word: OR
- Known as a disjunction

Inclusive Or (either or both)

If-Then Statements

- Symbol: \( \rightarrow \)
- Key Words: IF THEN
- Known as a conditional statement.
- In the statement If \( p \) then \( q \), \( p \) is the antecedent and \( q \) is the consequent.
- We say “\( p \) implies \( q \)”.

If and Only If Statements

- Symbol: \( \leftrightarrow \)
- Key Words: IF AND ONLY IF (sometimes abbreviated IFF)
- Known as a biconditional statement

Example: \( p \): I like apples. \( q \): You like peaches.

Write each statement in symbolic form.

a) I like apples and you like peaches.

b) I don't like apples or you like peaches.

c) If I like peaches, you like apples.

d) If I don't like apples, you don't like peaches.

e) I like apples if you don't like peaches.

f) I like apples if and only if you like peaches.

g) I don't like apples if you like peaches.

h) I like apples if you don’t like peaches.
p → q can be expressed in many different ways.


Consider the following statement: If I live in Destin, I live in Florida.
All of the following statements are equivalent and all are represented as p → q.

- I live in Florida if I live in Destin
- Living in Destin is sufficient for living in Florida.
- Living in Florida is necessary for living in Destin.
- I live in Destin only if I live in Florida.
- Only if I live in Florida, I live in Destin.
- I can extend this to the biconditional. If and only if is equivalent to “is necessary and sufficient”.

Parentheses when commas are used

- If a comma is used in the statement, necessary parentheses are placed around the elements on each side of the comma.

Example (page 133: 59–64) Write each compound statement in symbolic form.

p: The temperature outside is freezing.
q: The heater is working.
r: The house is cold.

59) The temperature outside is freezing and the heater is working, or the house is cold.

60) If the temperature outside is freezing, then the heater is working or the house is not cold.

61) If the temperature outside is freezing or the heater is not working, then the house is cold.

62) It is not the case that if the house is cold then the heater is not working.

63) The house is cold, if and only if the temperature outside is freezing and the heater isn’t working.

64) If the heater is working, then the temperature outside is freezing if and only if the house is cold.
Parentheses when commas are not used

- If a comma is not used in the statement, statements before and after the most dominant connective are grouped together.
  
  1. Biconditional  \( \leftrightarrow \)
  2. Conditional  \( \rightarrow \)
  3. And/Or  \( \land/\lor \)
  4. Negation  \( \sim \)

<table>
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<tr>
<th>Statement</th>
<th>Without Parentheses</th>
<th>Dominant connective</th>
<th>Symbolic Form</th>
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<tr>
<td>a) If the restaurant is closed then it is Monday or the parking lot is full.</td>
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<tr>
<td>b) The restaurant is not closed if and only if it is not Monday and the parking lot is not full</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>c) If the restaurant is closed or it is not Monday then the parking lot is not full</td>
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</table>

Example: Let p and q represent the following statements.

p: A student misses lecture.
q: A student studies.
r: A student fails.

Write each of the following symbolic statements in words.

a) \( p \lor q \)  

b) \( \sim (p \land q) \)  

c) \( \sim p \rightarrow q \)  

d) \( p \leftrightarrow r \)  

e) \( (p \land q) \rightarrow \sim r \)  

f) \( q \land (\sim p \rightarrow \sim r) \)
Example (page 133: 67–70). Write each symbolic statement in words.

p: The temperature is above 85 degrees.
q: We finished studying.
r: We go to the beach.

67) \((p \land q) \rightarrow r\)

68) \((q \land r) \rightarrow p\)

69) \(p \land (q \rightarrow r)\)

70) \(p \land (r \rightarrow q)\)

Summary of Symbols