The exponential law of uninhibited growth $(k > 0)$ or decay $(k < 0)$

\[ A = A_0 e^{kt}, \quad (k > 0) \]
\[ A = A_0 e^{kt}, \quad (k < 0) \]

\( A = A_0 e^{kt} \)
\( A_0 \) is the initial quantity of a substance present.
\( t \) is the length of time that has passed.
\( A \) is the amount of the substance that is present at time \( t \).
\( k \) is the growth or decay constant.

**Example 1**  The population of a colony of mosquitoes obeys the law of uninhibited growth. If there are 500 mosquitoes initially and there are 1200 after 2 days, what is the size after 5 days? How long will it be before there are 50,000 mosquitoes?

a. You are given that \( A_0 = 500 \) and \( A(t) = 1200 \) when \( t = 2 \).
   Substitute this information into \( A(t) = A_0 e^{kt} \) to find the growth constant \( k \).

b. Once you have found \( k \), you can substitute 5 for \( t \) in the equation \( A(t) = A_0 e^{kt} \) and find the value of \( A(5) \), the number of mosquitoes present after 5 days.

c. You can substitute 50,000 for \( A(t) \) in the equation \( A(t) = A_0 e^{kt} \) and solve for \( t \) to find the number of days it takes the mosquito population to reach 50,000.
Exponential Decay

**Example 2** A sample of 500 grams of radioactive lead-210 decays to 400 grams over a period of 7 years. Find the half-life of lead-210, the length of time it would take for the original 500 grams to decay to 200 grams.

a. You are given that \( A_0 = 500 \text{ g} \) and \( A(t) = 400 \text{ g} \) when \( t = 7 \text{ years} \).
   
   Substitute this information into \( A(t) = A_0 e^{kt} \) to find the decay constant \( k \).

b. To find the half-life, let \( A(t) = 250 \text{ g} \) (half of 500 g) and solve the equation \( A(t) = A_0 e^{kt} \) for \( t \).

Newton’s Law of Cooling

If the environment of an object is held at a constant temperature, Newton’s Law of Cooling states that the rate at which an object cools is directly proportional to the difference between its temperature and that of its environment. This results in an exponential decrease in the temperature of the object, which can be modeled by the following formula.

\[
u(t) = T + (u_0 - T)e^{kt}, \quad k < 0
\]

\( u(t) \) is the temperature of the object at time \( t \).
\( u_0 \) is the initial temperature of the object.
\( u_0 - T \) is the initial difference between temperature of the object and that of its environment.
\( T \) is the constant temperature of the environment.
**Example 3** A can of soda is warmed to room temperature (20° Celsius) and then placed in a refrigerator with a constant temperature of 3° Celsius. The soda can cools to a temperature of 14°C in 5 minutes.

a. Write a function to model this data.

b. Find the temperature of the can after 10 minutes.

c. How long will it take the can to reach a temperature of 8° C?

d. Describe the long-term behavior of the temperature of the can.
Logistic Growth Models

When growth of a quantity is unlimited a model such as the one just above to the left might be most appropriate. If there are constraints on the growth of a quantity such as the size of a population or even the size of a particular organism in that population, a logistic growth model may be more appropriate.

Such a function may be written in the form \( f(t) = \frac{a}{1 + be^{-rt}} \).

**Example 4** The population of a bacterial culture is growing according to the logistic growth model \( f(t) = \frac{10000}{1 + 19e^{-0.25t}} \) where \( t \) is the time elapsed in hours.

a. Find \( f(10) \), the population of the bacterial culture after 10 hours.

b. Graph the function using your calculator.

After \( y_1 = \), type \( 10000/(1 + 19e^{-0.25x}) \).

c. Also graph the horizontal line \( y = 8000 \), and find the time \( t \) when the population \( f(t) \) reaches 8000.

\[
f(t) = \frac{10000}{1 + 19e^{-0.25t}}
\]

d. Describe the long-term behavior of the population, the carrying capacity.