**General guidelines solving logarithmic equations**

If there are both logarithms and non-logarithmic terms in the equation, proceed as follows...

1. Combine all logarithmic expressions on one side and all constants on the other side of the equation.
2. Use logarithmic properties to rewrite the logarithmic expression as a single logarithm whose coefficient is 1. The form of the equation is \( \log_b M = N \).
3. Rewrite step 2 in exponential form \( b^N = M \) and solve the resulting equation for the variable.
4. Check each solution in the original equation, rejecting values that produce the logarithm of a negative number or the logarithm of 0.

If every term of the equation is a logarithm, condense each side of the equation to a single logarithm and use following useful fact:

Useful fact: If \( \log_b M = \log_b N \), \( (M > 0, N > 0) \), then \( M = N \).

(Special case \( b = e \)): If \( \ln M = \ln N \), \( (M > 0, N > 0) \), then \( M = N \).

**Example 1** Solve each equation.

a. \( \log_3 (x + 2) + \log_3 (2x - 5) = 4 \)

\[
\log_3 [(x + 2)(2x - 5)] = 4
\]

\[
(x + 2)(2x - 5) = 3^4
\]

\[
2x^2 - x - 10 = 81
\]

\[
2x^2 - x - 91 = 0
\]

\[
(x - 7)(2x + 13) = 0
\]

\[
x = 7 \quad \text{or} \quad x = -\frac{13}{2}
\]

\(-\frac{13}{2}\) is not a solution because it requires us to find the logarithm of a negative number.

\( \{7\} \) is the solution set.

b. \( \log_4 (2x - 2) + \log_4 (x + 3) = 3 \)

\[
\log_4 [(2x - 2)(x + 3)] = 3
\]

\[
(2x - 2)(x + 3) = 4^3
\]

\[
2x^2 + 4x - 6 = 64
\]

\[
2x^2 + 4x - 70 = 0
\]

\[
(x - 5)(2x + 14) = 0
\]

\[
x = 5 \quad \text{or} \quad x = -\frac{14}{2}
\]

\(-\frac{14}{2}\) is not a solution because it requires us to find the logarithm of a negative number.

\( \{5\} \) is the solution set.
Example 2  Solve each equation.

a. \( \ln(x - 2) - \ln(x + 3) = 2 \) 
\[
\ln\left(\frac{x-2}{x+3}\right) = 2 \\
e^{\ln\left(\frac{x-2}{x+3}\right)} = e^2 \\
\frac{x-2}{x+3} = e^2 \\
x - 2 = e^2(x + 3) \\
x - 2 = e^2x + 3e^2 \\
x - e^2x = 2 + 3e^2 \\
x(1 - e^2) = 2 + 3e^2 \\
x = \frac{2+3e^2}{1-e^2} \\
x \approx -3.7826 \quad \text{Be careful. } -3.7826 \text{ is not in the domain of the log expressions in the equation, so the solution set is } \{ \phi \}.

b. \( \ln(x + 4) - \ln(x - 1) = 3 \)

Example 3  Use a graphical method to solve the equation \( \log_2 x + \log_3 (4 - x) = 1 \).
Solving Exponential Equations

Example 4 Solve each equation.

a. \(3^x = 13\)
   \[
   \ln 3^x = \ln 13 \\
   x \ln 3 = \ln 13 \\
   x = \frac{\ln 13}{\ln 3} \\
   x \approx 2.3347
   \]

b. \(2^{x+1} = 3^{x-1}\)
   \[
   \ln 2^{x+1} = \ln 3^{x-1} \\
   (x + 1) \ln 2 = (x - 1) \ln 3 \\
   x \ln 2 + \ln 2 = x \ln 3 - \ln 3 \\
   -x \ln 3 - \ln 2 = -x \ln 3 - \ln 2 \\
   x = \frac{\ln 2 - \ln 3}{\ln 2 - \ln 3} \\
   x \approx 4.419
   \]

c. \(3e^{0.35t} = 6.6\)
   \[
   \frac{3e^{0.35t}}{3} = \frac{6.6}{3} \\
   e^{0.35t} = 2.2 \\
   \ln e^{0.35t} = \ln 2.2 \\
   0.35t = \ln 2.2 \\
   \frac{0.35t}{0.35} = \frac{\ln 2.2}{0.35} \\
   t \approx 2.2527
   \]

d. \(4^x = 47\)

e. \(3^{x+2} = 4^{x-3}\)

f. \(4e^{0.75t} = 21\)