8.3

\[
\begin{align*}
\log_{10} 0.9 & = 1.0 - 2.5 \\
\therefore 2.5 & = 1.0 - 0.9 \\
\end{align*}
\]
7. Expand the following logarithm as much as possible.

\[
\log \left( \frac{\sqrt{3x^2 + 7}}{x^3 (5x^2 + 2)^6} \right) = \log (3x^2 + 7)^{1/2} - \log (x^3)^{1/2} - \log (5x^2 + 2)^6.
\]
Solve each equation algebraically. Be sure to show your work.

10a. \( \log_4 (x+2) - \log_4 (x+6) = -1 \)

\[
\begin{align*}
\log_4 \left( \frac{x+2}{x+6} \right) &= -1 \\
\frac{x+2}{x+6} &= 4^{-1} \\
\frac{x+2}{x+6} &= \frac{1}{4} \\
4(x+2) &= x+6 \\
x+2 &= \frac{x+6}{4} \\
x &= 2
\end{align*}
\]

b. \( 3^{x-1} = 4 \)

\[
\begin{align*}
3^{x-1} &= 4 \\
\log_3 (3^{x-1}) &= \log_3 4 \\
x-1 &= \frac{\log_3 4}{1} \\
x &= 1 + \frac{\log_3 4}{1} \\
x &= 1 + \log_3 4
\end{align*}
\]
The vertices are \((0, a)\) and \((0, -a)\).

\[ \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \]

where \(0 < b < a\) and \(b^2 = a^2 - c^2\). Line:

\[ I = \frac{x^2}{c^2} + \frac{y^2}{a^2} \]

Major Axis along the \(y\)-axis.

Equation of an ellipse: Center at \((0, 0)\); Focus at \((0, \pm c)\);
Example 3: Find an equation of the ellipse with center (0, 0), one focus at (0, 4), and a vertex at (0, 5). Graph the equation.

\[ \frac{x^2}{9} + \frac{y^2}{25} = 1 \]

\[ b = \sqrt{25} = 5 \]

\[ c = 4 \]

\[ a = 5 \]
Example 4 Find the major and minor axes, focal, and vertices of the ellipse whose

\[ \frac{x^2}{9} + \frac{y^2}{4} = 1 \]
Example 5: Find an equation for the ellipse with center at (3, -2), one vertex at (-2, 2), one focus at (8, 2), and one point at (11.2, -2).
The hyperbola has two oblique asymptotes: \( ax + by = c \) and \( ax - by = c \). The equation of the asymptotes is given by:

\[
\frac{b^2}{a^2} = 1 \quad \frac{c^2}{a^2} = \frac{c^2}{b^2} - \frac{a^2}{c^2}
\]

The hyperbola that we will consider in this section will have transverse and conjugate axes, where the ellipse intersects its transverse axes.

The vertexes (plural of vertex) of a hyperbola are the points where the ellipse transverse axis.

The center of the hyperbola is the midpoint of the line segment joining the foci.

A hyperbola is the collection of all points in the plane the difference of whose distances from two fixed points, called the foci, is a constant.

MAC 1140 10.4 Hyperbolas
Example 1: Find an equation of the hyperbola with center (0, 0), one focus at (8, 0), and a vertex at (5, 0).

\[ \frac{x^2}{25} - \frac{y^2}{b^2} = 1 \]

Find the asymptotes. Graph the equation.

\[ a = 5 \]
\[ c = 8 \]
\[ b^2 = c^2 - a^2 = 8^2 - 5^2 = 39 \]

\[ b = \sqrt{39} \approx 6.2 \]

\[ 9 = \frac{a^2}{2} \]
\[ 2a = 2 \sqrt{9} = 6 \]

\[ a = 3 \]

\[ b = 3 \]

\[ 25 = 64 - 8 \]
\[ a = 8 \]
\[ b = 5 \]

\[ \frac{9}{2} = \frac{a}{2} - \frac{b}{2} \]
Example 2: Find the transverse and conjugate axes, foci, vertices, and asymptotes of the hyperbola whose equation is 

\[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1. \]
The hyperbola has two oblique asymptotes: 

\[ \frac{q}{p} = x \text{ and } \frac{q}{p} = x. \]

Asymptotes

The vertices are \((0, q)\) and \((0, -q)\).

\[ c^2 - a^2 = q^2 \quad \text{where} \quad c = \sqrt{\frac{q}{x} - \frac{c}{a}} \]

Transverse Axis along the Y-Axis.

Equation of a Hyperbola: Center at \((0, 0)\); Focal at \((0, \pm c)\).
Example 4. Find the transverse and conjugate axes, foci, vertices, and asymptotes of the hyperbola whose equation is \( \frac{x^2}{16} - \frac{y^2}{36} = 1 \). 

\[ c = \sqrt{a^2 + b^2} = \sqrt{16 + 36} = \sqrt{52} = 2 \sqrt{13} \]