Example 2: List the potential rational zeros of $p(x) = 6(x-1)(x-\frac{3}{2})(x+\frac{3}{5})$.

Then use your calculator to assist in finding them. Express $p(x)$ in factored form.

The Rational Zeros Theorem
Example 3
Find all zeros of \( p(x) = 3x^4 + x^3 - 3x^2 - 7x + 70 \)

Potential Rational Zeros

- \( \pm1, \pm2, \pm5, \pm7, \pm10, \pm14, \pm35, \pm70 \)

- \( \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm 7 \)

- \( \pm \frac{1}{3}, \pm \frac{5}{3}, \pm \frac{7}{3}, \pm 7 \)

- \( \pm \frac{1}{3}, \pm \frac{5}{3}, \pm \frac{7}{3}, \pm 7 \)

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- \( \pm \frac{1}{3}, \pm \frac{5}{3}, \pm \frac{7}{3}, \pm 7 \)
Linear Factorization Theorem

Every complex polynomial function \( f(x) \) of degree \( n \geq 1 \) can be factored into \( n \) linear factors (not necessarily distinct) of the form \( f(x) = a(x - r_1)(x - r_2) \cdots (x - r_n) \) where \( a \), \( r_1, r_2, \ldots, r_n \) are complex numbers. That is, every complex polynomial function of degree \( n \geq 1 \) has exactly \( n \) complex zeros, some of which may repeat.

It should be noted that the set of complex numbers includes the set of real numbers, so the complex zero which the Fundamental Theorem of Algebra guarantees may in fact be a real number.

\[ x^2 + 1 = 0 \]

\[ f(x) = (x - i)(x + i) \]

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The Fundamental Theorem of Algebra

If \( f(x) \) is a complex polynomial function of degree \( n \), where \( n \geq 1 \), then \( f \) has at least one complex zero.
Example 1: Find all zeros of the polynomial function \( f(x) = x^2 - 6x + 13 \) and write the polynomial expression in factored form. How many \( x \)-intercepts does the graph have?

\[
\begin{align*}
\frac{3 - \sqrt{3}}{2} + \frac{3 + \sqrt{3}}{2} &= \frac{3 + 3}{2} \\
&= 3 \\
\frac{2}{3} &= \frac{2}{3} \\
\sqrt{16} &= 4 \\
-16 &= 4
\end{align*}
\]
How many x-intercepts does the graph of this polynomial function have?

2 + 1 and 2 + i, 4, and -4.

Notice that the imaginary zeros appear in complex conjugate pairs.

For a total of two complex zeros.

The function has one real zero, 3, and four imaginary zeros: 2 + 1i, 2 - 1i, 4i, and -4i.

\[ f(x) = (x - 3)(x - 2)(x + 2)(x + 4) \]

Example 2 Find all zeros of the function \( f(x) = x^5 - 7x^4 + 33x^3 - 127x^2 + 272x - 240 \).
Example 3 Find a polynomial function with real coefficients, that has 4 and 3 + 2i as zeros.

Then is complex conjugate a -b is also a zero of p(x).

Let p(x) be a complex polynomial whose coefficients are real numbers. If a + bi is a zero of p(x), then is complex conjugate a -b is also a zero of p(x).

Example 3 Find a polynomial function with real coefficients, that has 4 and 3 + 2i as zeros.

Then is complex conjugate a -b is also a zero of p(x).

Let p(x) be a complex polynomial whose coefficients are real numbers. If a + bi is a zero of p(x), then is complex conjugate a -b is also a zero of p(x).

\[ p(x) = x^4 - 6x^2 + 9x - 4 \]

\[ p(-1) = (-1)^4 - 6(-1)^2 + 9(-1) - 4 = 1 - 6 - 9 - 4 = -16 \]

\[ p(x) = (x^2 - 3)(x - 3) + 2 \]

\[ f(x) = x^4 - 10x^2 + 37x - 52 \]
\[ P(x) = x^3 - 10x^2 + 37x - 52 \]

\[ P(x) = x^3 - 6x^2 + 13x - 4(x^2 - 6x + 13) \]
$f(x) = (x - (2 + \sqrt{3}))(x - (2 - \sqrt{3}))(x - (-6))(x - (-16))$

How many $x$-intercepts does its graph have?

Find two of its other zeros, and write such a polynomial function in factored form.

Example 4: A polynomial function $f(x)$ with real coefficients has zeros $6$ and $-16$. 
\[
\begin{align*}
\text{Example 5} & \quad \text{Give an example of a polynomial function whose only zeros are } 2, -1, \text{ and } 1 + 5i. \\
& \quad \text{Leave the answer in factored form. Will all coefficients of the polynomial necessarily be real?}
\end{align*}
\]
Example 6: Suppose a little bird tells you that 3 + i is one of the zeros of the function.

Find all of the other zeros of this function.

\[ f(x) = x^3 - 6x^2 + 18x + 20 x^2 + 30 \]

Zeros: 3 + i, 3 - i