STA 2023  Practice Problems for Exam 3

Provide an appropriate response.

1) Find the critical value $z_c$ that corresponds to a 95% confidence level.

$$z_{0.95} = \text{invNorm}(0.975, 0.1) \approx 1.96$$

2) Find the critical value $z_c$ that corresponds to a 99% confidence level.

$$z_{0.99} = \text{invNorm}(0.995, 0.1) \approx 2.56$$

3) A random sample of 120 students has a test score average with a standard deviation of 9.2. Find the margin of error if $c = 0.98$.

$$E = z_c \cdot \frac{s}{\sqrt{n}}$$

$$E = 2.33 \cdot \frac{9.2}{\sqrt{120}} \approx 1.96$$

4) A random sample of 150 students has a grade point average with a standard deviation of 0.78. Find the margin of error if $c = 0.98$.

$$\text{invNorm}(0.97, 0.1) = 2.33$$

$$E = z_c \cdot \frac{\sigma}{\sqrt{n}}$$

$$E = 2.33 \cdot \left(\frac{0.78}{\sqrt{150}}\right) \approx 0.15$$
5) A random sample of 40 students has a mean annual earnings of $3120 and a standard deviation of $677. Construct the confidence interval for the population mean, \( \mu \) if \( c = 0.95 \).

\[
E = z_{0.95} \cdot \frac{677}{\sqrt{40}} \approx 1.96
\]

\[
E = 1.96 \cdot \frac{677}{\sqrt{40}} \approx 209.8 \text{ or } 210
\]

\[
3120 - 210 < \mu < 3120 + 210
\]

\[
2910 < \mu < 3330
\]

6) A random sample of 56 fluorescent light bulbs has a mean life of 645 hours with a standard deviation of 31 hours. Construct a 95% confidence interval for the population mean.

\[
E = z_{0.95} \cdot \frac{31}{\sqrt{56}} \approx 8.1
\]

\[
645 - 8.1 < \mu < 645 + 8.1
\]

\[
636.9 < \mu < 653.1
\]

7) A group of 49 randomly selected students has a mean age of 22.4 years with a standard deviation of 3.8. Construct a 98% confidence interval for the population mean.

\[
E = z_{0.99} \cdot \frac{3.8}{\sqrt{49}} \approx 2.33
\]

\[
E = 2.33 \cdot \frac{3.8}{\sqrt{49}} \approx 1.26
\]

\[
22.4 - 1.26 < \mu < 22.4 + 1.26
\]

\[
21.14 < \mu < 23.66
\]

8) A nurse at a local hospital is interested in estimating the birth weight of infants. How large a sample must she select if she desires to be 99% confident that the true mean is within 2 ounces of the sample mean? The standard deviation of the birth weights is known to be 7 ounces.

\[
N = \left( \frac{z_{0.99} \sigma}{E} \right)^2 \approx 2.56
\]

\[
N = \left( \frac{2.56 \cdot 7}{2} \right)^2 \approx 80.3
\]
9) In order to fairly set flat rates for auto mechanics, a shop foreman needs to estimate the average time it takes to replace a fuel pump in a car. How large a sample must he select if he wants to be 99% confident that the true average time is within 15 minutes of the sample average? Assume the standard deviation of all times is 30 minutes.

\[ n = \left( \frac{Z_{0.01} \sigma}{E} \right)^2 \]
\[ Z_{0.99} = \text{invNorm}(0.995, 0, 1) \]
\[ n = \left( \frac{2.56(30)}{15} \right)^2 \approx 26.2 \Rightarrow 27 \]

10) Find the critical value, \( t_c \), for \( c = 0.95 \) and \( n = 16 \).

\[ d.f. = n - 1 = 16 - 1 = 15 \]
\[ t_{0.95} = \text{invT}(0.975, 15) \]
\[ t_{0.95} \approx 2.13 \]

11) Find the critical value, \( t_c \) for \( c = 0.99 \) and \( n = 10 \).

\[ d.f. = n - 1 = 10 - 1 = 9 \]
\[ t_{0.99} = \text{invT}(0.995, 9) \]
\[ t_{0.99} \approx 3.25 \]

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

12) In a random sample of 28 families, the average weekly food expense was $95.60 with a standard deviation of $22.50. Determine whether a normal distribution or a t-distribution should be used or whether neither of these can be used to construct a confidence interval. Assume the distribution of weekly food expenses is normally shaped.

A) Use normal distribution.
B) Use the t-distribution.
C) Cannot use normal distribution or t-distribution.

This is a small sample because \( n = 28 \leq 30 \).

The normal distribution cannot be used for this small sample because \( \sigma \) is not known.

The t-distribution can be used because the population is normally distributed.
13) For a sample of 20 IQ scores the mean score is 105.8. The standard deviation, \( \sigma \), is 15. Determine whether a normal distribution or a t-distribution should be used or whether neither of these can be used to construct a confidence interval. Assume that IQ scores are normally distributed.

A) Cannot use normal distribution or t-distribution.
B) Use normal distribution.
C) Use the t-distribution.

The normal distribution can be used because the standard deviation of the population is known and because the population is normally distributed.

14) A random sample of 15 statistics textbooks has a mean price of $105 with a standard deviation of $30.25. Determine whether a normal distribution or a t-distribution should be used or whether neither of these can be used to construct a confidence interval. Assume the distribution of statistics textbook prices is not normally distributed.

A) Use normal distribution.
B) Cannot use normal distribution or t-distribution.
C) Use the t-distribution.

Since the sample is small and the population of textbook prices is not normally distributed, neither the normal distribution nor the t-distribution can be used.

15) Find the value of \( E \), the margin of error, for \( c = 0.99 \), \( n = 10 \) and \( s = 3.2 \).

\[
E = t_{0.99} \cdot \frac{s}{\sqrt{n}}
\]

\[
t_{0.99} = \text{invT}(0.995, 9)
\]

\[
t_{0.99} \approx 3.25
\]

\[
E = 3.25 \cdot \frac{3.2}{\sqrt{10}}
\]

\[
E \approx 3.29
\]
16) Construct a 95% confidence interval for the population mean, $\mu$. Assume the population has a normal distribution. A sample of 20 college students had mean annual earnings of $3120 with a standard deviation of $677.

$$t_{0.95} = 2.09, \ d.f. = n-1 = 20-1 = 19$$

$$E = t_{0.95} \cdot \frac{S}{\sqrt{n}}$$

$$\bar{x} - E < \mu < \bar{x} + E$$

$$E = 2.09 \cdot \frac{677}{\sqrt{20}}$$

$$3120 - 316 < \mu < 3120 + 316$$

$$2804 < \mu < 3436$$

$$E \approx 316$$

17) Construct a 90% confidence interval for the population mean, $\mu$. Assume the population has a normal distribution. A sample of 15 randomly selected students has a grade point average of 2.86 with a standard deviation of 0.78.

$$t_{0.90} = \text{invT}(0.95,14) \approx 1.76$$

$$E = t_{0.90} \cdot \frac{S}{\sqrt{n}}$$

$$E = 1.76 \cdot \frac{0.78}{\sqrt{15}} \approx 0.354$$

$$2.86 - 0.354 < \mu < 2.86 + 0.354$$

$$2.506 < \mu < 3.214$$

18) When 435 college students were surveyed, 120 said they own their car. Find a point estimate for $p$, the population proportion of students who own their cars.

$$\hat{p} = \frac{120}{435} = 0.276$$

19) A survey of 300 fatal accidents showed that 123 were alcohol related. Construct a 98% confidence interval for the proportion of fatal accidents that were alcohol related.

$$E = z_{0.99} \sqrt{\frac{\hat{p} \cdot (1-\hat{p})}{n}}$$

$$z_{0.99} = \text{invNorm}(0.99,0,1) \approx 2.33$$

$$E = 2.33 \sqrt{\frac{0.41 \cdot 0.59}{300}}$$

$$\hat{p} = \frac{123}{300} \approx 0.41$$

$$0.41 - 0.066 < \hat{p} < 0.41 + 0.066$$

$$0.344 < \hat{p} < 0.476$$
20) When 495 college students were surveyed, 150 said they own their car. Construct a 95% confidence interval for the proportion of college students who say they own their cars.

\[ E = z_{0.95} \sqrt{\frac{\hat{p} \cdot q}{n}} = \frac{z_{0.95}}{\sqrt{\frac{0.303 \cdot 0.697}{495}}} = 0.040 \]

\[ \hat{p} = \frac{150}{495} = 0.303 \]

\[ q = 1 - 0.303 = 0.697 \]

\[ 0.303 - 0.040 < \hat{p} < 0.303 + 0.040 \]

\[ 0.263 < \hat{p} < 0.343 \]

21) A researcher at a major hospital wishes to estimate the proportion of the adult population of the United States that has high blood pressure. How large a sample is needed in order to be 95% confident that the sample proportion will not differ from the true proportion by more than 4%?

\[ n = \hat{p} \cdot q \left( \frac{z_{0.95}}{E} \right)^2 \]

\[ z_{0.95} = \text{invNORM}(0.975, 0) = 1.96 \]

\[ n = (0.5) \cdot (0.5) \left( \frac{1.96}{0.04} \right)^2 \]

\[ n = 600 \cdot 0.25 \approx 150 \]

22) The mean age of bus drivers in Chicago is 48.5 years. Write the null and alternative hypotheses.

\[ H_0: \mu = 48.5 \text{ years} \]

\[ H_a: \mu \neq 48.5 \text{ years} \]

23) The mean IQ of statistics teachers is greater than 110. Write the null and alternative hypotheses.

\[ H_0: \mu \leq 110 \]

\[ H_a: \mu > 110 \]
24) A candidate for governor of a particular state claims to be favored by at least half of the voters. Write the null and alternative hypotheses.

\[ H_0 : \pi \geq 0.5 \]
\[ H_a : \pi < 0.5 \]

25) The mean score for all NBA games during a particular season was less than 101 points per game. Write the null and alternative hypotheses.

\[ H_0 : \mu \geq 101 \]
\[ H_a : \mu < 101 \]

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

26) Given \( H_0 : \pi \geq 80\% \) and \( H_a : \pi < 80\% \), determine whether the hypothesis test is left-tailed, right-tailed, or two-tailed.

A) two-tailed      B) right-tailed      C) left-tailed

The inequality for \( H_a \) points to the left.

27) Given \( H_0 : \mu \leq 25 \) and \( H_a : \mu > 25 \), determine whether the hypothesis test is left-tailed, right-tailed, or two-tailed.

A) two-tailed      B) left-tailed      C) right-tailed

The inequality for \( H_a \) points to the right.
28) A researcher claims that 62% of voters favor gun control. Determine whether the hypothesis test for this claim is left-tailed, right-tailed, or two-tailed:
   A) right-tailed
   B) two-tailed
   C) left-tailed

\[ H_0: p = 0.62 \]
\[ H_a: p \neq 0.62 \]

29) The mean age of bus drivers in Chicago is 52.5 years. Identify the type I and type II errors for the hypothesis test of this claim.

\[ H_0: \mu = 52.5 \text{yr} \]
\[ H_a: \mu < 52.5 \]

Type I: \( \mu = 52.5 \text{yr} \) is true, but we reject \( H_0 \).

Type II: \( \mu = 52.5 \text{yr} \) is false, but we fail to reject \( H_0 \).

30) The mean IQ of statistics teachers is greater than 120. Identify the type I and type II errors for the hypothesis test of this claim.

\[ H_0: \mu \leq 120 \]
\[ H_a: \mu > 120 \]

Type I: \( \mu \leq 120 \) is true, but we reject \( H_0 \).

Type II: \( \mu \leq 120 \) is false, but we fail to reject \( H_0 \).

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

31) The mean age of bus drivers in Chicago is 50.2 years. If a hypothesis test is performed, how should you interpret a decision that rejects the null hypothesis?
   A) There is not sufficient evidence to support the claim \( \mu = 50.2 \).
   B) There is sufficient evidence to reject the claim \( \mu = 50.2 \).
   C) There is not sufficient evidence to reject the claim \( \mu = 50.2 \).
   D) There is sufficient evidence to support the claim \( \mu = 50.2 \).

\[ H_0: \mu = 50.2 \text{ years (Claim)} \]
\[ H_a: \mu \neq 50.2 \text{ years} \]

Since the claim is \( H_0 \), we either reject the claim or fail to reject it. Since the decision rejects \( H_0 \), we say there is enough evidence to reject the claim.
32) The mean age of bus drivers in Chicago is greater than 57.8 years. If a hypothesis test is performed, how should you interpret a decision that rejects the null hypothesis?

\[ H_0 : \mu \leq 57.8 \]
\[ H_a : \mu > 57.8 \text{ (claim)} \]
When the claim is \( H_a \), we either support it or fail to support it.
Since the decision rejects \( H_0 \), we say there is enough evidence to support the claim.

A) There is sufficient evidence to support the claim \( \mu > 57.8 \).
B) There is not sufficient evidence to reject the claim \( \mu > 57.8 \).
C) There is sufficient evidence to reject the claim \( \mu > 57.8 \).
D) There is not sufficient evidence to support the claim \( \mu > 57.8 \).

33) The mean age of bus drivers in Chicago is greater than 47.6 years. If a hypothesis test is performed, how should you interpret a decision that fails to reject the null hypothesis?

\[ H_0 : \mu \leq 47.6 \]
\[ H_a : \mu > 47.6 \text{ (claim)} \]
When the claim is \( H_a \), we either support it or fail to support it.
Since the decision fails to reject \( H_0 \), we say there is not enough evidence to support the claim.

A) There is sufficient evidence to support the claim \( \mu > 47.6 \).
B) There is not sufficient evidence to reject the claim \( \mu > 47.6 \).
C) There is not sufficient evidence to reject the claim \( \mu > 47.6 \).
D) There is not sufficient evidence to support the claim \( \mu > 47.6 \).

34) A candidate for governor of a certain state claims to be favored by at least half of the voters. If a hypothesis test is performed, how should you interpret a decision that fails to reject the null hypothesis?

\[ H_0 : p \geq 0.5 \text{ (claim)} \]
\[ H_a : p < 0.5 \]
When the claim is \( H_a \), we either reject it or fail to reject it.
Since the decision fails to reject \( H_0 \), we say there is not enough evidence to reject the claim.

A) There is sufficient evidence to support the claim \( p \geq 0.5 \).
B) There is not sufficient evidence to reject the claim \( p \geq 0.5 \).
C) There is sufficient evidence to reject the claim \( p \geq 0.5 \).
D) There is not sufficient evidence to support the claim \( p \geq 0.5 \).

35) Suppose you are using \( \alpha = 0.05 \) to test the claim that \( \mu > 14 \) using a P-value. You are given the sample statistics \( n = 50, \bar{x} = 14.3, \) and \( s = 1.2 \). Find the P-value.

\[ H_0 : \mu \leq 14 \]
\[ H_a : \mu > 14 \]
\[ Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{14.3 - 14}{1.2/\sqrt{50}} \approx 1.77 \]

\[ P = \text{normalcdf}(1.77, 6, 0, 1) \approx 0.038 \]
Since \( P < \alpha \), we reject \( H_0 \).
36) Suppose you are using $\alpha = 0.05$ to test the claim that $\mu \neq 14$ using a $P$-value. You are given
the sample statistics $n = 35$, $x = 13.1$, and $s = 2.7$. Find the $P$-value.

\[ Z = \frac{13.1 - 14}{2.7/\sqrt{35}} \approx -1.97 \]

\[ P = \text{normalcdf}(0, -1.97) \approx 0.049 \]

There is sufficient evidence to support the claim.

37) Given $H_0: \mu > 85$ and $P = 0.007$. Do you reject or fail to reject $H_0$ at the 0.01 level of significance?

A) reject $H_0$
B) not sufficient information to decide
C) fail to reject $H_0$

\[ P = 0.007 \]

\[ \alpha = 0.01 \]

\[ P < \alpha \]

38) Given $H_0: \mu = 25$, $H_a: \mu \neq 25$, and $P = 0.034$. Do you reject or fail to reject $H_0$ at the 0.01 level of significance?

A) not sufficient information to decide
B) fail to reject $H_0$
C) reject $H_0$

\[ P = 0.034 \]

\[ \alpha = 0.01 \]

\[ P \geq \alpha \]

39) Find the critical value for a right-tailed test with $\alpha = 0.01$ and $n = 75$. 

\[ \alpha = 0.01 \]

\[ Z = \text{invNorm}(0.99, 0.1) \approx 2.33 \]
40) Find the critical value for a two-tailed test with \( \alpha = 0.01 \) and \( n = 30 \).

\[ z_0 = \pm 2.58 \]

41) Find the critical value for a left-tailed test with \( \alpha = 0.05 \) and \( n = 48 \).

\[ z_0 = \text{invNorm}(0.05, 0, 1) = -1.645 \]

42) Test the claim that \( \mu > 19 \), given that \( \alpha = 0.05 \) and the sample statistics are \( n = 50 \), \( \bar{x} = 19.3 \), and \( s = 1.2 \).

\[ H_0: \mu = 19 \]
\[ H_a: \mu > 19 \text{ (claim)} \]

\[ Z = \frac{19.3 - 19}{1.2/\sqrt{50}} = 1.77 \]

This is in the rejection region, so we reject \( H_0 \).

There is sufficient evidence to support the claim that \( \mu > 19 \).

43) Test the claim that \( \mu \neq 38 \), given that \( \alpha = 0.05 \) and the sample statistics are \( n = 35 \), \( \bar{x} = 37.1 \) and \( s = 2.7 \).

\[ H_0: \mu = 38 \]
\[ H_a: \mu \neq 38 \text{ (claim)} \]

\[ Z = \frac{37.1 - 38}{2.7/\sqrt{35}} \]

\[ Z = -1.97 \]

Reject \( H_0 \).

There is just barely enough evidence to support the claim.
44) A trucking firm suspects that the mean lifetime of a certain tire it uses is less than 34,000 miles. To check the claim, the firm randomly selects and tests 54 of these tires and gets a mean lifetime of 33,390 miles with a standard deviation of 1200 miles. At $\alpha = 0.05$, test the trucking firm's claim.
1) ±1.96  
2) ±2.575  
3) 1.96  
4) 0.15  
5) ($2910, $3330)  
6) (636.9, 653.1)  
7) (21.1, 23.7)  
8) 82  
9) 27  
10) 2.131  
11) 3.250  
12) B  
13) B  
14) B  
15) 3.29  
16) ($2803, $3437)  
17) (2.51, 3.21)  
18) 0.276  
19) (0.344, 0.476)  
20) (0.263, 0.344)  
21) 601  
22) H₀: μ = 48.5, Hₐ: μ ≠ 48.5  
23) H₀: μ ≤ 110, Hₐ: μ > 110  
24) H₀: p ≥ 0.5, Hₐ: p < 0.5  
25) H₀: μ ≥ 101, Hₐ: μ < 101  
26) C  
27) C  
28) B  
29) type I: rejecting H₀: μ = 52.5 when μ = 52.5  
    type II: failing to reject H₀: μ = 52.5 when μ ≠ 52.5  
30) type I: rejecting H₀: μ ≤ 120 when μ ≤ 120  
    type II: failing to reject H₀: μ ≤ 120 when μ > 120  
31) B  
32) A  
33) D  
34) B  
35) 0.0384  
36) 0.0488  
37) A  
38) B  
39) 2.33  
40) ±2.575  
41) -1.645  
42) standardized test statistic = 1.77; critical value = 1.645; reject H₀; There is enough evidence to support the claim.  
43) standardized test statistic = -1.97; critical value = ±1.96; reject H₀; There is enough evidence to support the claim.  
44) standardized test statistic = -3.74; critical value z₀ = -1.645; reject H₀; There is sufficient evidence to support the trucking firm's claim.