MAC 1140 Review Problems from Section 5.8

1. A colony of bacteria undergoes uninhibited growth as indicated by the formula $A(t) = 2000e^{0.20t}$.
   a. What is the initial number of bacteria present?
   b. How long does it take the colony of bacteria to double its population?

2. The half-life of radioactive iodine 131 is 8.0 days.
   a. If $A(t) = A_0e^{kt}$ gives the amount of iodine 131 left after $t$ days, find $k$.
   b. If 3.00 g of Iodine 131 is initially present, how much time must pass before only 0.15 g of Iodine 131 remains?

3. Strontium 90 is a radioactive isotope that was spewed (spewn?) by accidents at Three Mile Island & Chernobyl.
   Suppose 100 g of Strontium 90 is initially present, and the amount present after $t$ years is given by $A(t) = 100e^{kt}$.
   If 100 g of Strontium 90 takes 4.25 years to decay to 90 g, find the decay constant $k$, and find the half life of Strontium 90.

4. A thermometer reading $35^\circ$ C is brought into a room with a constant temperature of $20^\circ$ C. If the thermometer reads $30^\circ$ C after 5 minutes, a. what will it read after being in the room for 10 minutes? b. how long must it be in the room before it reads $22^\circ$ C?

5. The population of bacteria in a particular petri dish is approximated by the logistic model $P(t) = \frac{15000}{1 + 19e^{-0.37t}}$ where the time $t$ is given in hours.
   a. Graph this function over the domain $0 \leq t \leq 24$.
   b. What is the carrying capacity of the petri dish?
   c. Time $t = 0$ corresponds to 8:00 a.m. on March 12. What is the population of bacteria in the dish at 8:00 a.m. on March 12?
   d. What is the population of bacteria at 6:00 p.m. on March 12?
   e. At what time is the population estimated to reach 14000?
Solutions

1. a. \( A(0) = 2000e^{0.20(0)} \)
   \[ A(0) = 2000e^{0} \]
   \[ A(0) = 2000(1) \]
   \[ A(0) = 2000 \]

   b. \( 4000 = 2000e^{0.20t} \)
   \[ \frac{4000}{2000} = e^{0.20t} \]
   \[ 2 = e^{0.20t} \]
   \[ \ln 2 = \ln e^{0.20t} \]
   \[ \ln 2 = 0.20t \]
   \[ t = \frac{\ln 2}{0.20} \approx 3.4657 \text{ hours} \]

2. a. \( 0.5A_0 = A_0e^{k(8 \text{ days})} \)
   \[ 0.5 = e^{k(8 \text{ days})} \]
   \[ \ln 0.5 = \ln e^{k(8 \text{ days})} \]
   \[ \ln 0.5 = k(8 \text{ days}) \]
   \[ \frac{\ln 0.5}{(8 \text{ days})} = k(8 \text{ days}) \]
   \[ k \approx -0.0866/\text{day} \]

   b. \( 0.15 = (3.00)e^{(-0.0866/\text{day})(t)} \)
   \[ \frac{0.15}{3.00} = \frac{(3.00)e^{(-0.0866/\text{day})(t)}}{3.00} \]
   \[ 0.05 = e^{(-0.0866/\text{day})(t)} \]
   \[ \ln 0.05 = \ln e^{(-0.0866/\text{day})(t)} \]
   \[ \ln 0.05 = (-0.0866/\text{day})t \]
   \[ t = \frac{\ln 0.05}{-0.0866/\text{day}} \approx 34.6 \text{ days} \]

3. a. \( A(4.25) = 100e^{k(4.25 \text{ years})} \)
   \[ 90 = 100e^{(4.25 \text{ years})k} \]
   \[ 0.9 = e^{(4.25 \text{ years})k} \]
   \[ \ln 0.9 = \ln e^{(4.25 \text{ years})k} \]
   \[ \ln 0.9 = (4.25 \text{ years})k \]
   \[ k = \frac{\ln 0.9}{4.25 \text{ years}} \approx -0.0248/\text{year} \]

   b. \( A(t) = 100e^{(-0.0248/\text{year})(t)} \)
   \[ 50 = 100e^{(-0.0248/\text{year})(t)} \]
   \[ \frac{1}{2} = e^{(-0.0248/\text{year})(t)} \]
   \[ \ln \frac{1}{2} = \ln e^{(-0.0248/\text{year})(t)} \]
   \[ \ln \frac{1}{2} = (-0.0248/\text{year})t \]
   \[ t = \frac{\ln \frac{1}{2}}{-0.0248/\text{year}} \approx 28 \text{ years} \]
4. Use the function \( u(t) = T + (u_0 - T)e^{kt} \) with \( u_0 = 35^\circ, T = 20^\circ, \) and the fact that \( u(5) = 30^\circ. \)

\[ \begin{align*}
30^\circ &= 20^\circ + (35^\circ - 20^\circ)e^{k(5 \text{ min})} \\
30^\circ &= 20^\circ + (15^\circ)e^{k(5 \text{ min})} \\
10^\circ &= (15^\circ)e^{k(5 \text{ min})} \\
\frac{10^\circ}{15^\circ} &= (15^\circ)e^{k(5 \text{ min})} \\
\frac{2}{3} &= e^{k(5 \text{ min})} \\
\ln\left(\frac{2}{3}\right) &= \ln\left(e^{k(5 \text{ min})}\right) \\
\ln\left(\frac{2}{3}\right) &= k(5 \text{ min}) \\
\frac{\ln\left(\frac{2}{3}\right)}{5 \text{ min}} &= \frac{k(5 \text{ min})}{5 \text{ min}} \\
k &\approx -0.0811/\text{min}
\end{align*} \]

a. \( u(t) = 20^\circ + (35^\circ - 20^\circ)e^{-0.0811t} \)

\[ \begin{align*}
\text{u(10)} &= 20^\circ + (35^\circ - 20^\circ)e^{-0.0811(10)} \\
\text{u(10)} &\approx 26.7^\circ
\end{align*} \]

b. \( u(t) = 20^\circ + (35^\circ - 20^\circ)e^{-0.0811t} \)

\[ \begin{align*}
22^\circ &= 20^\circ + (35^\circ - 20^\circ)e^{-0.0811t} \\
2^\circ &= 15^\circ e^{-0.0811t} \\
\ln\left(\frac{2}{15}\right) &= \ln e^{-0.0811t} \\
-0.0811t &= \ln\left(\frac{2}{15}\right) \\
t &\approx 24.845 \text{ minutes}
\end{align*} \]

5. The population of bacteria in a particular petri dish is approximated by the logistic model \( P(t) = \frac{15000}{1 + 19e^{-0.37t}} \) where the time \( t \) is given in hours.

a. \( Y_1 = 15000/(1 + 19e^{(-.37x)}) \)

\[ \begin{align*}
\text{Xmin} &= 0, \quad \text{Xmax} = 24, \quad \text{Xscl} = 1, \quad \text{Ymin} = 0, \quad \text{Ymax} = 20000, \quad \text{Yscl} = 5000.
\end{align*} \]

For reference, you might also graph the line \( Y_2 = 15000. \)

This is the horizontal asymptote.

b. As \( t \to +\infty, \ P(t) \to 15000. \) So the carrying capacity is 15000.

c. \( P(0) = \frac{15000}{1 + 19e^{0}} = \frac{15000}{1 + 19e^{0}} = \frac{15000}{1 + 19(1)} = \frac{15000}{1 + 19} = \frac{15000}{20} = 750 \)

d. 6:00 p.m. is 10 hours after 8:00 a.m., so \( t = 10. \)

\[ P(10) = \frac{15000}{1 + 19e^{-0.37(10)}} \approx 10200. \]

e. \( P(t) = 14000 \)

\[ \frac{15000}{1 + 19e^{-0.37t}} = 14000 \]

You could solve this algebraically if you wanted to, but I will be content with the graphical solution if you can find it that way.

\( t \approx 15.09 \) hours or about 15 hours and 5 minutes.

So, the population of bacteria is expected to reach 14000 at about 11:05 p.m.