Rewrite each inequality so that 0 is on the right-hand side and a single polynomial or rational expression is on the left-hand side. Construct a sign graph for the expression on the left-hand side and use this information to write the solution set to the inequality.

1. \(2x^4 + 9x^3 - 19x - 12 > 0\)
2. \(\frac{2x+5}{x-4} \leq 0\)
3. \(\frac{2x+5}{x-4} \leq 3\)
4. \(\frac{12}{x^2 - 6x + 2} \geq -2\)

5. Find the domain of the function \(f(x) = \sqrt{\frac{(x + 3)(4 - x)}{(x + 1)^2}}\).

Solutions

1. \(2x^4 + 9x^3 - 19x - 12 > 0\)

To construct a sign graph for the expression \(2x^4 + 9x^3 - 19x - 12\), we need to find its zeros. Its potential rational zeros would be a good place to begin our search.

\[ \frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{3}{2} \]

Upon inspection of the graph it appears that \(-4, -1, \text{ and } \frac{3}{2}\) are all zeros of \(2x^4 + 9x^3 - 19x - 12\). Furthermore, \(-1\) appears to have multiplicity 2, since the graph merely touches the \(x\)-axis at \(x = -1\) rather than crossing it.

The leading coefficient of \(2x^4 + 9x^3 - 19x - 12\) is 2, so we can factor it as follows:

\[2(x + 4)(x + 1)^2 \left(x - \frac{3}{2}\right) \quad \text{or} \quad (x + 4)(x + 1)^2(2x - 3)\]

The sign graph can be obtained by inspecting the graph of \(y = (x + 4)(x + 1)^2(2x - 3)\) using your calculator. Simply place the zeros \(-4, -1, \text{ and } \frac{3}{2}\) on the number line and observe whether the function is positive (if the graph is above the \(x\)-axis) or negative (if the graph is below the \(x\)-axis) on each interval. The sign graph looks like this:

\(< + + + + (-4) --- - (-1) --- - \left(-\frac{3}{2}\right) + + + + >\)

The solution set to the inequality \(2x^4 + 9x^3 - 19x - 12 > 0\) is the union of the intervals where we have pluses.

The solution set is \((-\infty, -4) \cup \left(\frac{3}{2}, \infty\right)\).

The endpoints \(-4\) and \(\frac{3}{2}\) are excluded from the intervals because the inequality is strict.
2. \( \frac{2x + 5}{x - 4} \leq 0 \)

The numbers to be placed on the number line for the sign graph are the zeros of the numerator and the zeros of the denominator.

\[
\begin{align*}
2x + 5 &= 0 & x - 4 &= 0 \\
x &= -\frac{5}{2} & x &= 4
\end{align*}
\]

The sign graph can be obtained by inspecting the graph of \( y = \frac{2x + 5}{x - 4} \) using your calculator. Simply place the zeros \(-\frac{5}{2}\) and 4 on the number line and observe whether the function is positive (if the graph is above the \( x \)-axis) or negative (if the graph is below the \( x \)-axis) on each interval. The sign graph looks like this:

\(< \quad ++++\left( -\frac{5}{2} \right) \quad --- \quad - (4) \quad +++ >\)

The solution set consists of the interval where \( \frac{2x + 5}{x - 4} \leq 0 \), so it is the interval with the minuses. It is the interval \([-\frac{5}{2}, 4)\). It includes the endpoint \(-\frac{5}{2}\) because the expression is equal to 0 at \(-\frac{5}{2}\). It excludes the endpoint 4 because the expression is undefined at \( x = 4 \).

3. \( \frac{2x + 5}{x - 4} \leq 3 \)

\[
\begin{align*}
\frac{2x + 5}{x - 4} - 3 &\leq 0 \\
\frac{2x + 5}{x - 4} - \frac{3(x - 4)}{x - 4} &\leq 0 \\
\frac{2x + 5 - 3(x - 4)}{x - 4} &\leq 0 \\
\frac{2x + 5 - 3x + 12}{x - 4} &\leq 0 \\
\frac{-x + 17}{x - 4} &\leq 0
\end{align*}
\]

The numbers to be placed on the sign graph are 17, the zero of the numerator, and 4, the zero of the denominator. By graphing the equation \( y = \frac{-x + 17}{x - 4} \) and observing whether the function is positive (if the graph is above the \( x \)-axis) or negative (if the graph is below the \( x \)-axis) on each interval, we obtain the following sign graph.

\(< \quad --- \quad -(4) \quad +++ \quad +(17) \quad -- -- >\)

The solution set includes the intervals where \( \frac{-x + 17}{x - 4} \) is negative together with any endpoints that make it zero. The solution set is \((-\infty, 4) \cup [17, \infty)\).

The 4 is excluded from the solution set because the expression is undefined at \( x = 4 \). The 17 is included because the expression \( \frac{-x + 17}{x - 4} \) is equal to 0 at \( x = 17 \).
4. \[
\frac{12}{x^2 - 6x + 2} \geq -2 \\
\frac{12}{x^2 - 6x + 2} + 2 \geq 0 \\
\frac{12}{x^2 - 6x + 2} + \frac{2(x^2 - 6x + 2)}{x^2 - 6x + 2} \geq 0 \\
\frac{12 + 2(x^2 - 6x + 2)}{x^2 - 6x + 2} \geq 0
\]
\[
\frac{12 + 2x^2 - 12x + 4}{x^2 - 6x + 2} \geq 0 \\
\frac{2x^2 - 12x + 16}{x^2 - 6x + 2} \geq 0 \\
\frac{2(x^2 - 6x + 8)}{x^2 - 6x + 2} \geq 0 \\
\frac{2(x - 2)(x - 4)}{x^2 - 6x + 2} \geq 0
\]

The numbers to be placed on the sign graph are the zeros of the numerator and the zeros of the denominator.

\[2(x - 2)(x - 4) = 0 \quad x^2 - 6x + 2 = 0 \] (Use the quadratic formula.)

\[x = 2 \quad \text{or} \quad x = 4 \quad \text{x} = 3 + \sqrt{7} \approx 5.65 \quad \text{or} \quad x = 3 - \sqrt{7} \approx 0.35 \]

Place the numbers \(3 - \sqrt{7}, 2, 4, \) and \(3 + \sqrt{7}\) in order from left to right on the real number line. Then inspect the graph of \(y = \frac{2(x - 2)(x - 4)}{(x^2 - 6x + 2)}\) to see whether the expression \(\frac{2(x - 2)(x - 4)}{x^2 - 6x + 2}\) is positive or negative on each interval. Here is the resulting sign graph.

\[< + + + +(3 - \sqrt{7}) -- - (2) +++ + (4) -- - - (3 + \sqrt{7}) +++ + >\]

The solution set includes the intervals where the expression \(\frac{2(x - 2)(x - 4)}{x^2 - 6x + 2}\) is positive, together with any endpoints that make it 0.

The solution set is \((-\infty, 3 - \sqrt{7}) \cup [2, 4] \cup (3 + \sqrt{7}, \infty)\).

The numbers \(3 - \sqrt{7}\) and \(3 + \sqrt{7}\) are excluded because the expression is undefined at these \(x\)-values. The numbers 2 and 4 are included because the expression has a value of 0 at these \(x\)-values.

5. Find the domain of the function \(f(x) = \sqrt{\frac{(x + 3)(4 - x)}{(x + 1)^2}}\).

The domain of \(f\) is the set of real numbers for which \(\frac{(x + 3)(4 - x)}{(x + 1)^2} \geq 0\).

The numbers to be placed on the number line for the sign graph are \(-3, -1, \) and 4.

By inspecting the graph of \(y = \frac{(x + 3)(4 - x)}{(x + 1)^2}\) we can obtain the following sign graph: \(< -- - (-3) +++ + (-1) +++ + (4) -- - - >\)

The solution set includes the intervals that make the expression \(\frac{(x + 3)(4 - x)}{(x + 1)^2}\) positive, together with any endpoints that make it 0.

The solution set is \([-3, -1) \cup (-1, 4]\).

The numbers \(-3\) and 4 are included because the expression \(\frac{(x + 3)(4 - x)}{(x + 1)^2}\) has a value of 0 at these \(x\)-values. The number \(-1\) is excluded because the expression \(\frac{(x + 3)(4 - x)}{(x + 1)^2}\) is undefined at \(-1\).

So, the domain of the function \(f\) is \([-3, -1) \cup (-1, 4]\).