MAC 1140 Solutions to the Review Problems from Chapter 12

Solutions

1. \( a_1 = (1)^2 + 4 = 5 \)
   \( a_2 = (2)^2 + 4 = 8 \)
   \( a_3 = (3)^2 + 4 = 13 \)
   \( a_4 = (4)^2 + 4 = 20 \)
   \( a_5 = (5)^2 + 4 = 29 \)

2. \( a_1 = 1 \)
   \( a_2 = 2a_1 + 1 = 2(1) + 1 = 3 \)
   \( a_3 = 2a_2 + 1 = 2(3) + 1 = 7 \)
   \( a_4 = 2a_3 + 1 = 2(7) + 1 = 15 \)
   \( a_5 = 2a_4 + 1 = 2(15) + 1 = 31 \)

3. 

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<thead>
<tr>
<th>( n )</th>
<th>( 4n )</th>
<th>( a_n = 4n + 1 )</th>
<th>( \frac{n}{2n+1} )</th>
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<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>( \frac{1}{3} )</td>
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<td>2</td>
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<td>( \frac{2}{5} )</td>
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<td>3</td>
<td>12</td>
<td>13</td>
<td>3</td>
<td>( \frac{3}{7} )</td>
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<td>4</td>
<td>16</td>
<td>17</td>
<td>4</td>
<td>( \frac{4}{9} )</td>
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<td>3</td>
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4. Evaluate each sum using a calculator if necessary.
   a. \( \sum_{i=1}^{4} i^3 = 100 \)
   b. \( \sum (2k^2 - 3k - 7) = 30 \)
   c. \( \sum_{n=0}^{n} \frac{1}{n!} \approx 2.718279 \)

5. Use the formulas \( \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \) and \( \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \)
   to rewrite each sum as a simplified polynomial in the variable \( n \).
   a. \( \sum_{k=1}^{n} (12k + 4) \)
   \( \sum_{k=1}^{n} 12k + \frac{4n}{n} \)
   \( 12 \frac{n(n+1)}{2} + 4n \)
   \( 6n(n+1) + 4n \)
   \( 6n^2 + 6n + 4n \)
   \( 6n^2 + 10n \)
   b. \( \sum_{k=1}^{n} (12k^2 - 8k + 3) \)
   \( \sum_{k=1}^{n} 12k^2 - \frac{8n}{n} + \frac{3n}{n} \)
   \( 12 \frac{n(n+1)(2n+1)}{6} - 8 \frac{n(n+1)}{2} + 3n \)
   \( 2n(n+1)(2n+1) - 4n(n+1) + 3n \)
   \( 2n(2n^2 + 3n + 1) - (4n^2 + 4n) + 3n \)
   \( 4n^3 + 6n^2 + 2n - 4n^2 - 4n + 3n \)
   \( 4n^3 + 2n^2 + n \)
6. The 19th term of an arithmetic sequence is 123 and the 49th term is 303. Find a nonrecursive expression for the nth term.

\[ d = \frac{a_n - a_m}{n - m} = \frac{a_{49} - a_{19}}{49 - 19} = \frac{303 - 123}{30} = 6 \]
\[ a_n = a_m + (n - m)d \]
\[ a_n = a_{19} + (n - 19)(6) \]
\[ a_n = 123 + 6n - 114 \]
\[ a_n = 6n + 9 \]

7. Find the sum of the terms of each finite arithmetic sequence:

a. \[ 18 + 25 + 32 + \ldots + 102 = \]
\[ d = 25 - 18 = 7 \]
\[ a_n = a_1 + (n - 1)d \]
\[ a_n = 18 + (n - 1)(7) \]
\[ a_n = 7n + 11 \]
Which term is 102?
\[ 7n + 11 = 102 \]
\[ 7n = 91 \]
\[ n = 13 \]
102 is the 13th term.
\[ S_n = 13\left(\frac{a_1 + a_{13}}{2}\right) = 13\left(\frac{18 + 102}{2}\right) = 13\left(\frac{120}{2}\right) = 13(60) = 780 \]

b. \[ 9 + 7 + 5 + \ldots + (-105) + (-107) = \]
\[ d = 7 - 9 = -2 \]
\[ a_n = a_1 + (n - 1)(-2) \]
\[ a_n = 9 + (n - 1)(-2) \]
\[ a_n = -2n + 11 \]
Which term is -107?
\[ -2n + 11 = -107 \]
\[ -2n = -118 \]
\[ n = 59 \]
-107 is the 59th term.
\[ S_n = 59\left(\frac{a_1 + a_{59}}{2}\right) = 59\left(\frac{9 + (-107)}{2}\right) = 59\left(\frac{-98}{2}\right) = 59(-49) = -2891 \]

8. The first term of a geometric sequence is 150 and the common ratio is \( \frac{3}{5} \).

a. List the first 5 terms. 150, 90, 54, 32.5, 19.44
b. Find a nonrecursive expression for the nth term.
\[ a_n = a_1 r^{n-1} \]
\[ a_n = 150\left(\frac{3}{5}\right)^{n-1} \]
c. Find the sum of the first 10 terms.
\[ S_n = a_1 \left(\frac{1 - r^n}{1 - r}\right) \]
\[ S_{10} = a_1 \left(\frac{1 - \left(\frac{3}{5}\right)^{10}}{1 - \frac{3}{5}}\right) = 150 \left(\frac{1 - \frac{59049}{9765625}}{1 - \frac{3}{5}}\right) = \frac{29119728}{78125} \approx 372.7325184 \]
\[ S_{10} = \sum_{n=1}^{10} 150\left(\frac{3}{5}\right)^{n-1} \]
\[ S_{10} = \text{sum(seq}(150(3/5)^{(n-1)}, n, 1, 10, 1)) \approx 372.7325184 \]
d. Find the sum of all of the terms of this sequence.
\[ S = \sum_{n=1}^{\infty} 150\left(\frac{3}{5}\right)^{n-1} = \frac{150}{1 - \frac{3}{5}} = \frac{150}{\frac{2}{5}} = 150 \cdot \frac{5}{2} = 375 \]
9. Contribution made by the deposit at the end of the 60th month: $200
Contribution made by the deposit at the end of the 59th month: $200(1.00375)
Contribution made by the deposit at the end of the 58th month: $200(1.00375)^2
Contribution made by the deposit at the end of the 1st month: $200(1.00375)^{59}
The total balance is the sum of the following finite geometric series:
$200 + 200(1.00375) + 200(1.00375)^2 + \ldots + 200(1.00375)^{59}$.
It has $n = 60$ terms and the common ratio is $r = 1.00375$.
So the sum is given by the formula $S_n = a_1 \left(\frac{1-r^n}{1-r}\right)$.
$S_{60} = 200 \left(\frac{1-(1.00375)^{60}}{1-1.00375}\right) = 13429.11$

10. Find the value of each geometric series.

a. $\sum_{i=1}^{\infty} \frac{3}{10^i} = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \ldots = \frac{\frac{3}{10}}{1 - \frac{1}{10}} = \frac{3}{9} = \frac{1}{3}$

b. $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$

c. $100 + 60 + 36 + \ldots = \frac{100}{1 - 0.6} = \frac{100}{0.4} = 250$

11. a. $\frac{8!}{3! \cdot 5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3 \cdot 2 \cdot 1 \cdot 5!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$

b. $\frac{20!}{17!} = \frac{20 \cdot 19 \cdot 18 \cdot 17!}{17!} = 20 \cdot 19 \cdot 18 = 6840$

12. $(2x - 5)^5 = 32x^5 - 400x^4 + 2000x^3 - 5000x^2 + 6250x - 3125$

13. Find the coefficient of the $x^9y^7$ term in the expansion of $(2x - y)^{16}$.

\[
\binom{16}{7} (2x)^9(-y)^7 = -5857280x^9y^7
\]

The coefficient of the $x^9y^7$ term is $-5857280$.

You really wouldn't want to do the entire expansion just to find that one coefficient.

After combining like terms, the expansion looks something like this.

$(2x - y)^{16} = 65536x^{16} - 524288y)x^{15} + 1966080y^2x^{14} - 4587520y^3x^{13} + 7454720y^4x^{12} - 8945664y^5x^{11} + 8200192y^6x^{10} - 5857280y^7x^9 + 3294720y^8x^8 - 1464320y^9x^7 + 512512y^{10}x^6 - 139776y^{11}x^5 + 29120y^{12}x^4 - 4480y^{13}x^3 + 480y^{14}x^2 - 32y^{15}x + y^{16}$