22nd ANNUAL

NORTHWEST FLORIDA STATE COLLEGE

MATHEMATICS TOURNAMENT

WRITTEN TEST

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Niceville, Florida

Test Booklet

INSTRUCTIONS: **<u>This is a 90 minute, 45 problem, multiple-choice examination.</u>** There are five (5) possible responses to each question or problem. You are to **<u>select the one (1) best answer to each</u>.** You may mark on the test booklet, and the back of each page may be used for additional work space. Darken completely the circle below the letter of your response to each question on your score sheet. Your student number is encoded on your score sheet for you. Mark your answers **boldly** with a No. 2 pencil. If you must change an answer, completely erase your first choice and then record the new answer. Incomplete erasures and multiple marks for any question will be scored as an incorrect response. <u>Do not mark beyond question 45</u>. Your score will be computed by the following formula: Score = 45 + (4C - I) , where C represents the number of correct answers and I represents the number of incorrect answers. If you can definitely rule out at least one choice, it will be in your favor to randomly guess from the remaining choices. There is no penalty for problems left unanswered. In the event of a tie, the indicated tie-breaker questions will be checked in order until the tie is broken.

Review and check your score sheet carefully. Your student identification number has been encoded on your answer sheet and it has been checked by our marked-sense card reader. If you alter this number in any way you may **<u>disqualify yourself and your team from consideration for any awards</u>.**

When you complete your test, close your test booklet, turn your answer sheet over, and sit quietly until all of the answer sheets are collected. You may keep your pencil and your test booklet. **Calculators are Not Allowed!**

PLEASE DO NOT OPEN
UNTIL INSTRUCTED TO DO SO

1. $\dfrac{2\sqrt{2}-i^3}{1-2i\sqrt{2}} = ?$

   A. $-i$

   B. $-2\sqrt{2}+i$

   C. $2\sqrt{2}-i$

   D. i

   E. $\dfrac{\sqrt{2}-i}{3}$

2. How many pairs of natural numbers (m,n) satisfy the equation $\dfrac{4}{m}+\dfrac{2}{n}=1$?

   A. 1

   B. 2

   C. 3

   D. 4

   E. More than 4

3. **(Tie Break No. 1)** If the surface area of a closed rectangular box is 11 and the sum of the lengths of its 12 edges is 24. What is the length of its diagonal?

   A. $2\sqrt{3}$

   B. $\sqrt{14}$

   C. 5

   D. 7

   E. 6

4. The line l passes through the point $(-2,0)$ and intersects the circle $x^2 + y^2 = 2x$ at two points. The range of the slope m of the line l is

A. $(-2\sqrt{2}, 2\sqrt{2})$

B. $(-\sqrt{2}/4, \sqrt{2}/4)$

C. $(-\sqrt{2}, \sqrt{2})$

D. $(-1/2, 1/2)$

E. $(-\sqrt{2}/2, \sqrt{2}/2)$

5. Find the period of the function $f(x) = \sin\frac{x}{2} + \cos\frac{x}{3}$.

A. $\pi/6$

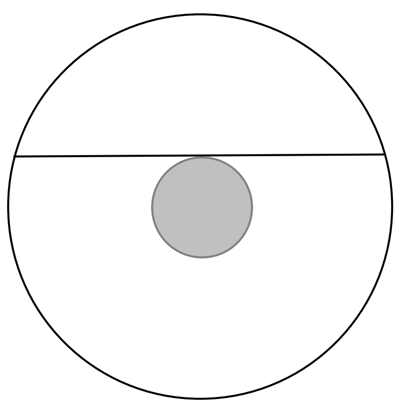
B. 2π

C. 6π

D. 12π

E. 24π

6. The circles in the figure are concentric. The chord is tangent to the inner circle and has length 14. What is the exact area of the non-shaded region?



A. 49π

B. 7π

C. 14π

D. $7\pi^2$

E. None of the above

7. $\{a_k\}$ is an arithmetic sequence. $a_1 = 18$ and the common difference $d = -3$. Find all n such that the partial sum $S_n = \sum_{k=1}^{n} a_k$ has the maximum value.

A. 6 or 7

B. 6

C. 7

D. 5 or 6

E. None of the above

8. **(Tie Break No. 2)** In the figure, the length of the sides of the square *ABCD* is 8. $OA = 7$ and $OB = 3$. Find the area of the triangle ΔOCD.

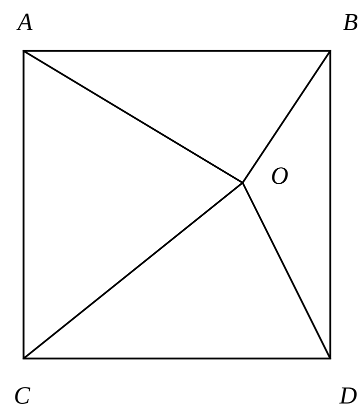
A. $6\sqrt{3}$

B. 12

C. $32 - 6\sqrt{3}$

D. 15

E. $9\sqrt{3}$



9. A circle passes though the points $(0, 0)$ and $(2, 4)$. Its center is on the line $2x - y = 5$. Find the equation of the circle.

A. $x^2 + y^2 - 4x - 3y = 0$

B. $x^2 + y^2 - 6x - 2y = 0$

C. $x^2 + y^2 - 2x - 4y = 0$

D. $x^2 + y^2 + 2x - 6y = 0$

E. $x^2 + y^2 + 4x - 7y = 0$

10. For what values of a and b are the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \ (a > 0, b > 0)$ perpendicular?

A. $a = 2,\ b = 1/2$

B. $a = b$

C. $a = 2b$

D. $a = 5,\ b = \sqrt{5}$

E. None of the above

11. A total of 120 numbers can be formed using all five digits 1, 2, 3, 4, and 5. If these numbers are arranged in increasing order (12345, 12354, 12435, 12453 up to 54321) which one is 75th in this order?

A. 35421

B. 42531

C. 41235

D. 41325

E. 53421

12. If the three sides $a, b,$ and c of triangle ΔABC are also the first three consecutive terms of a geometric sequence and $c = 2a$, find $\cos B$

A. $1/4$

B. $3/4$

C. $\sqrt{2}/4$

D. $\sqrt{2}/3$

E. $1/3$

13. Solve $|x^2 - 2x - 16| = 8$.

A. $\{-4, 0, 4\}$

B. $\{-4, -2, 4, 6\}$

C. $\{-6, -4, 2, 4\}$

D. $\{-4, 4\}$

E. No Solution

14. The area of the solution region of the system of the inequalities $\begin{Bmatrix} y \geq x - 1 \\ y \leq -3|x| + 1 \end{Bmatrix}$ is

A. $\sqrt{2}$

B. $3/2$

C. $3\sqrt{2}/2$

D. 2

E. None of the above

15. Find the real solutions of the equation $\sqrt{\log x - 3} = \log x - 3$.

A. {0,1}

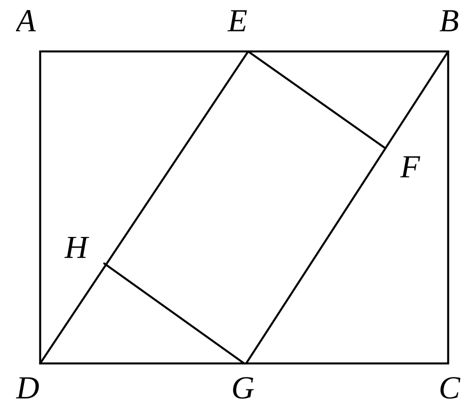
B. {10,3}

C. {1,000, 10,000}

D. {0.001, 0.0001}

E. No Solution

16. **(Tie Break No. 3)** In rectangle *ABCD*, $AB = CD = 4$ and $AD = CB = 3$. Points *E* and *G* are the midpoints of *AB* and *CD* respectively. $EF \perp GB$ and $GH \perp DE$. Find the area of rectangle *EFGH* shown in the figure.



A. $48/13$

B. $54/13$

C. $60/13$

D. $66/13$

E. None of the above

17. Find all values of *x* where the graph of $f(x) = \dfrac{3x^2 + 5x - 1}{x^2 - x + 3}$ intersects its horizontal asymptote.

A. $x = 5/4$

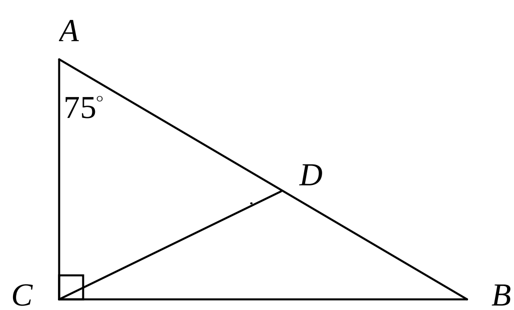
B. $x = 3$

C. $x = 4/5$

D. $x = 4/5$ or $x = 3$

E. There is no intersection

18. ΔABC is a right triangle with $\angle A = 75^\circ$ and $\angle C = 90^\circ$. Point *D* is on side *AB*. If $AD = CD$ and $AC = 10$, find the area of ΔCBD.



A. $25\left(2-\sqrt{3}\right)$

B. $25\left(1-\sqrt{3}\right)$

C. $25\left(1+\sqrt{3}\right)$

D. $25\left(2+\sqrt{3}\right)$

E. None of the above

19. If $f(x) = x^2 - 2$ and $g(x) = x - 3$, find all values of x such that $(f \circ g)(x) = (g \circ f)(x)$

A. $x = 3$

B. $x = 0$

C. $x = 2$

D. $x = 4$

E. None of the above

20. Solve the equation $a^2 = \log_b a^3$ for a, given $b = a^5$ and a and b are real numbers.

A. $a = 5$

B. $a = \dfrac{\sqrt{3}}{3}$

C. $a = \dfrac{\sqrt{15}}{5}$

D. $a = 3$

E. None of the above

21. The base of a triangle is 40 feet and one of the base angles is 60°. The sum of the other two sides is 45 feet. What is the measure of the shortest side of the triangle?

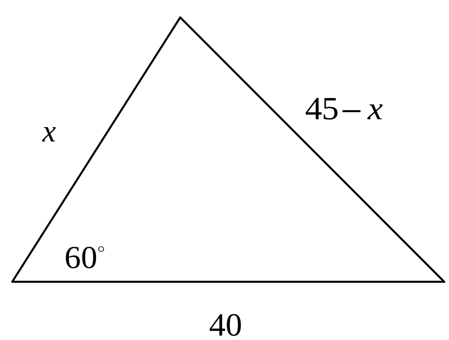
A. 22.5

B. 20

C. 18

D. 8.5

E. 6



22. A particular sphere and a particular right circular cylinder have the same volume. If the radius of the cylinder is 6 times its height, find the ratio of the surface area of the sphere to the surface area of the cylinder.

A. $2/7$

B. $3/7$

C. $4/7$

D. $5/7$

E. None of the above

23. Find the domain of the function $f(x) = 1 + \dfrac{1}{1 + \dfrac{1}{\ln x}}$.

A. $\left(0, \frac{1}{e}\right) \cup \left(\frac{1}{e}, 1\right) \cup (1, \infty)$

B. $(0, \infty)$

C. $(0, 1) \cup (1, \infty)$

D. $(0, 1) \cup (1, e) \cup (e, \infty)$

E. None of the above

24. A hiker starts at noon at the top of a mountain 2 miles high, and walks down a straight path to the bottom at a rate of 5 miles per hour. Another hiker starts at noon at the bottom of same mountain and walks up the same path at a rate of 3 miles per hour. The straight path is 16 miles long. What altitude are the hikers at when they meet?

A. $1/2$ mile

B. 1 mile

C. $4/3$ mile

D. $2/3$ mile

E. $3/4$ mile

25. Solve the inequality $\frac{10}{x^2 - x - 12} > -1$

A. $(-\infty, -3) \cup (-1, -\infty)$

B. $(-3, -1) \cup (2, 4)$

C. $(-\infty, -3) \cup (4, \infty)$

D. $(-\infty, -3) \cup (-1, 2) \cup (4, \infty)$

E. None of the above

26. Suppose that a rope surrounds the earth at the equator. The rope is lengthened by 10 *ft*. By how much is the rope raised above the earth?

A. 2π *ft*.

B. $\frac{10}{\pi}$ *ft*.

C. $\frac{5}{\pi}$ *ft*.

D. π *ft*.

E. 1 *ft*.

27. **(Tie Break No. 4)** A man is digging a hole and standing in it. He is 5 *ft* 10 *in* tall. He says that he is one fourth done. When he is finished, the top of his head will be 3 times as far below the ground as it is now above ground. How deep will the hole be when finished?

A. 14 *ft* 3 *in*

B. 13 *ft* 4 *in*

C. 12 *ft* 7 *in*

D. 14 *ft* 0 *in*

E. 13 *ft* 2 *in*

28. Maximize $P(x, y) = 5x + 9y$ subject to $x \geq 0$, $y \geq 0$, and $x + 2y \leq 6$.

A. 27

B. 0

C. 30

D. 45

E. 14

29. After an initial deposit of *x* dollars, the amount of money in a certain fund is doubled at the end of each month for 5 months. If at the end of the 5-month period there is a total of $560 in the fund, how much money was in the fund at the beginning of the third month?

A. $17.50

B. $35

C. $70

D. $140

E. $224

30. In the expression $\frac{x}{2} + \frac{2}{x} + \frac{x}{2} + \frac{2}{x} + \ldots$ if each odd-numbered term is $\frac{x}{2}$ and each even-numbered term is $\frac{2}{x}$, then what is the sum of the first 56 terms of the expression?

A. $\frac{14x^2 + 14}{x}$

B. $\frac{28x^2 + 28}{x}$

C. $\frac{7x^2 + 7}{x}$

D. $\frac{7x^2 + 32}{x}$

E. $\frac{14x^2 + 56}{x}$

31. If $2 \le x \le 4$ and $2 \le y \le 4$, then the maximum possible value of $x - \frac{x}{y}$ is

A. 1

B. $3/2$

C. 2

D. 3

E. $7/2$

32. In the figure, the right triangle *ABC* has $\angle C = 90^\circ$ and contains a point *D*. If $AD = 10$, $CD = 6$, and $\angle ADC = \angle CDB = \angle BDA$. Find BD.

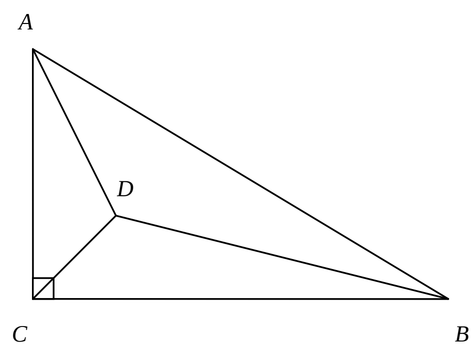
A. $30\sqrt{3}$

B. 28

C. 39

D. 33

E. $30\sqrt{2}$



33. Given a rectangle, if one side is decreased by 3 and the adjacent side is increased by 2, it forms a square with area 25. What is the perimeter of the rectangle?

A. 20

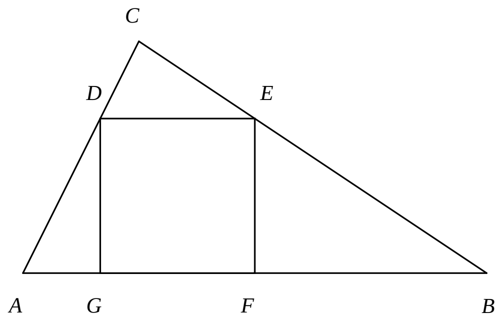
B. 16

C. 12

D. 22

E. None of the above

34. In the figure, the right triangle *ABC* has $\angle C = 90^\circ$, $BC = a$, and $AC = b$. The square *DEFG* is inscribed in the triangle. Find the perimeter of the square.



A. $\dfrac{4ab\sqrt{a^2+b^2}}{a^2-ab+b^2}$

B. $\dfrac{4\sqrt{a^2+b^2}}{a^2+ab+b^2}$

C. $\dfrac{4ab\sqrt{a^2+b^2}}{a^2+ab+b^2}$

D. $\dfrac{4\sqrt{a^2+b^2}}{a^2-ab+b^2}$

E. None of the above

35. Find $\lim\limits_{n\to\infty} \dfrac{\left(1+\frac{1}{2}+\frac{1}{2^2}+\cdots+\frac{1}{2^n}\right)2^n}{1+3\cdot 2^n}$.

A. 0

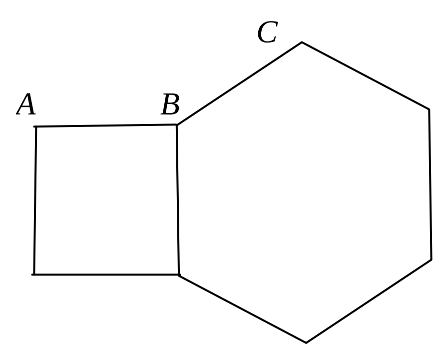
B. 1

C. $2/3$

D. $1/3$

E. 2

36. Point *B* is a mutual vertex of a regular hexagon, a square and a third regular polygon. If two of the sides of this third polygon are AB and BC, what is this polygon?



A. An octagon

B. A heptagon

C. A dodecagon

D. A decagon

E. None of the above

37. **(Tie Break No. 5)** Which of the following trig expressions is identical to $\dfrac{\cos(A-B)}{\sin^2 B-\cos^2 A}$?

A. $-\sec(A+B)$

B. $\cot(A-B)$

C. $\sec(A+B)$

D. $-\cot(A-B)$

E. $\sec(A-B)$

38. In a triangle ΔABC $\sin A+\cos A=\dfrac{\sqrt{2}}{2}$, $AC=4$, and $AB=3$, find the area of the triangle.

A. $3\sqrt{2}$

B. $3\sqrt{2+\sqrt{3}}$

C. $3\sqrt{2-\sqrt{3}}$

D. $3\sqrt{3}$

E. Not enough information given

39. Which of following sine curves has amplitude 3, period $\pi/2$, phase shift $\pi/4$, and vertical shift 1?

A. $3\sin(4x-\pi)+1$

B. $3\sin(4x+\pi)-1$

C. $3\sin\left(2x-\dfrac{\pi}{2}\right)+1$

D. $3\sin\left(2x+\dfrac{\pi}{2}\right)-1$

E. $3\sin(4x-\pi)-1$

40. Find the exact value of $\tan 112.5^\circ$.

A. $\sqrt{\dfrac{2+\sqrt{2}}{2-\sqrt{2}}}$

B. $-\sqrt{\dfrac{2-\sqrt{2}}{2+\sqrt{2}}}$

C. $\sqrt{2}-2$

D. $-1+\sqrt{2}$

E. $-1-\sqrt{2}$

41. Solve the inequality $0 < \sin x + \cos x < 1$ for $0 \le x < 2\pi$

A. $(0,\ 3\pi/4) \cup (7\pi/4,\ 2\pi)$

B. $(2\pi/3,\ 3\pi/4) \cup (11\pi/6,\ 2\pi)$

C. $(\pi/2,\ 3\pi/4) \cup (7\pi/4,\ 2\pi)$

D. $(\pi/2,\ 3\pi/4) \cup (11\pi/6,\ 2\pi)$

E. None of the above

42. Given that $f(2) = 5$ and $f'(x) = 2xf(x)$, find the value of $g'(\pi/3)$ if $g(x) = f(\sec x)$.

A. $2\pi/9$

B. $5\sqrt{3}$

C. $10\pi/\sqrt{3}$

D. $40\sqrt{3}$

E. $5\pi\sqrt{3}/9$

43. Find $\cos\theta$ if $\cos 2\theta = -\frac{1}{9}$ and $\frac{\pi}{2} \le 2\theta \le \pi$.

A. $\frac{2\sqrt{2}}{3}$

B. $\frac{\sqrt{2}}{3}$

C. $-\frac{2}{3}$

D. $\frac{2}{3}$

E. None of the above

44. Find the sum of all solutions of $\sin 2\theta - 2\sin\theta = 0$, where $0 \le \theta \le 2\pi$.

A. 3π

B. 4π

C. 5π

D. 6π

E. 7π

45. Seven people sit on one row of ten seats. How many different ways can the three empty seats be distributed if there are no two empty seats next to each other and the first and last seats must be occupied?

A. 20

B. 35

C. 56

D. 84

E. None of the above