21st ANNUAL

MATHEMATICS TOURNAMENT

WRITTEN TEST

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Test Booklet

INSTRUCTIONS: **This is a 90 minute, 45 problem, multiple-choice examination.** There are five (5) possible responses to each question or problem. You are to **select the one (1) best answer to each.** You may mark on the test booklet, and the back of each page may be used for additional work space. Darken completely the circle below the letter of your response to each question on your score sheet. Your student number is encoded on your score sheet for you. Mark your answers **boldly** with a No. 2 pencil. If you must change an answer, completely erase your first choice and then record the new answer. Incomplete erasures and multiple marks for any question will be scored as an incorrect response. Do not mark beyond question 45. Your score will be computed by the following formula: Score = 45 + (4C - I) , where C represents the number of correct answers and I represents the number of incorrect answers. If you can definitely rule out at least one choice, it will be in your favor to randomly guess from the remaining choices. There is no penalty for problems left unanswered. In the event of a tie, the indicated tie-breaker questions will be checked in order until the tie is broken.

Review and check your score sheet carefully. Your student identification number has been encoded on your answer sheet and it has been checked by our marked-sense card reader. If you alter this number in any way you may **disqualify yourself and your team from consideration for any awards.**

When you complete your test, close your test booklet, turn your answer sheet over, and sit quietly until all of the answer sheets are collected. You may keep your pencil and your test booklet. **Calculators are Not Allowed!**

PLEASE DO NOT OPEN
UNTIL INSTRUCTED TO DO SO

Written Test

1. If the average of two numbers is $2x+1$ and one of the numbers is x, what is the other number?

   A. $x+1$

   B. $2x-1$

   C. $3x+1$

   D. $3x+2$

   E. None of the above

2. Find the perimeter of a regular hexagon inscribed in a circle of radius 5 meters.

   A. 30 meters

   B. 15 meters

   C. 5 meters

   D. $5p$ meters

   E. $15p$ meters

3. The two shortest sides of a right triangle have lengths 2 and $\sqrt{5}$ respectively. Let x be the smallest angle of the triangle. What is $\cos x$?

   A. $\frac{\sqrt{5}}{9}$

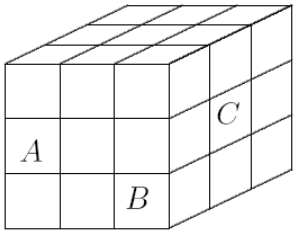
   B. $\frac{2}{9}$

   C. $\frac{2}{3}$

   D. $\frac{\sqrt{5}}{3}$

   E. $\frac{2}{\sqrt{5}}$

4. The figure below shows a solid cube with edge length 3. What is the surface area of the figure obtained by removing the three labeled unit cubes from the large cube shown?



A. 50

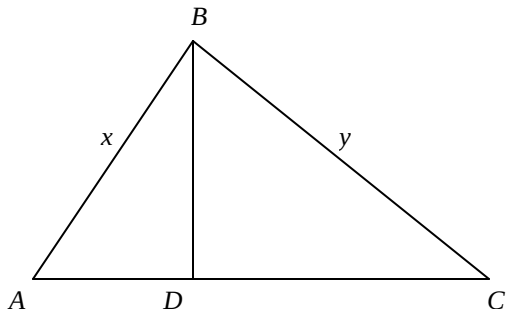
B. 54

C. 60

D. 64

E. 66

5. In the figure, triangle *ABC* is a right triangle with right angle *B*. *BD* is an altitude of the triangle. If $AB = x$ and $BC = y$, find the ratio of the area of triangle *ABD* to the area of triangle *BCD*.



A. $\frac{x}{y}$

B. $\frac{x^2}{y^2}$

C. $\frac{y^2}{\sqrt{x^2+y^2}}$

D. $\frac{x^2}{\sqrt{x^2+y^2}}$

E. None of the above.

6. Hurricanes are very low pressure areas with diameters of over 500 miles. The barometric air pressure "*P*" in inches of mercury at a distance of "*x*" miles from the eye of a hurricane is modeled by the function $P(x) = 0.5\log_9(x+1) + 27$. At what distance from the eye of the hurricane is the air pressure 28 inches of mercury?

A. 57 miles

B. 65 miles

C. 80 miles

D. 97 miles

E. 112 miles

7. Let P be the point $(3, 2)$. Let Q be the reflection of P about the x-axis. Let R be the reflection of Q about the line $y = -x$, and let S be the reflection of R about the origin. What is the distance between points P and S?

A. $\sqrt{52}$

B. $\sqrt{26}$

C. $\sqrt{50}$

D. 4

E. 13

8. The diameter of a circle is 28. An equilateral triangle is inscribed in the circle. What is the perimeter of the triangle?

A. $42\sqrt{3}$

B. $28\sqrt{3}$

C. 42

D. 70

E. None of the above

9. A cup of coffee and a cup of cream are sitting side by side on a counter, and they contain equal volumes of liquid. Bob removes $1/10$ of the cream from its cup and adds it to the cup of coffee. Then he removes $1/10$ of this mixture and adds it to the cup of cream. What portion of the mixture in the cream cup is coffee?

A. $\frac{1}{10}$

B. $\frac{1}{100}$

C. $\frac{1}{101}$

D. $\frac{10}{101}$

E. $\frac{100}{101}$

10. A regular octagon stop sign is to be made from a square piece of sheet metal. The largest stop sign that can be cut from the square has sides of length s. What is the area of the square?

A. $(2+\sqrt{3})s^2$

B. $9s^2$

C. $(2+3\sqrt{2})s^2$

D. $(3+2\sqrt{2})s^2$

E. None of the above

11. The volume "*V*" of a sphere with radius "*r*" is given by $V(r)=\frac{4}{3}pr^3$, and the surface area "*S*" is given by $S(r)=4pr^2$. Find $V(S)$.

A. $\frac{2r}{3p}S$

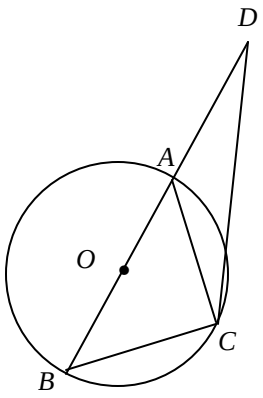
B. $\frac{3}{2p}S^{\frac{3}{2}}$

C. $\frac{4}{p}S^{\frac{3}{2}}$

D. $\frac{p}{6}\left(\frac{S}{p}\right)^{\frac{3}{2}}$

E. $\frac{3}{p}\left(\frac{S}{4}\right)^{\frac{3}{2}}$

12. In the given figure, the center *O* of the circle is on the segment *BD*. If $AD=AC$ and $\angle D=15^\circ$. Find the degree measure of $\angle B$.



A. 45°

B. 30°

C. 60°

D. 75°

E. 15°

13. The ratio of an interior angle of a regular polygon to an exterior angle is 8 to 1. How many sides does the polygon have?

A. 12

B. 14

C. 16

D. 18

E. None of the above

14. Find the value of *m* so that the line $y = mx$ divides the region enclosed by $y = 2x - x^2$ and the *x*-axis into two parts that have equal area.

A. $\frac{1}{3}$

B. $\sqrt[3]{4}$

C. $2 + \sqrt[3]{4}$

D. $2 - \sqrt[3]{4}$

E. $\sqrt[3]{6}$

15. Let $f(x) = \frac{3x-7}{5x-4}$. Find $f^{-1}(x)$

A. $\frac{5x-4}{3x-7}$

B. $\frac{4x-7}{3-5x}$

C. $\frac{-3x+7}{5x-4}$

D. $-\frac{3x-7}{5x-4}$

E. $\frac{4x-7}{5x-3}$

16. How many subsets does $S = A \cup B$ have if $A = \{-1, 0, 1, 2\}$ and $B = \{1, 2, 5\}$?

A. 8

B. 13

C. 32

D. 64

E. 7

17. The values of y that will satisfy the system of equations

$$2x^2 + 6x + 5y + 1 = 0$$
$$2x + y + 3 = 0$$

may be found by solving which of the following:

A. $y^2 + 14y - 7 = 0$

B. $y^2 + 8y + 1 = 0$

C. $y^2 + 10y - 7 = 0$

D. $y^2 + y - 12 = 0$

E. None of the above

18. $\cos^{-1}\left(\cos\frac{5p}{4}\right) + \sin^{-1}\left(\sin\frac{5p}{4}\right) = ?$

A. $\frac{p}{2}$

B. $\frac{3p}{2}$

C. $\frac{5p}{2}$

D. $\frac{7p}{2}$

E. None of the above

19. A circle of radius 6 has half of its area removed by cutting away a border of uniform width. Find the width of the border.

A. $64\sqrt{3}$

B. $6-3\sqrt{2}$

C. $2\sqrt{3}$

D. 3

E. None of the above

20. Find the sum of the solutions to the equation $\sin x = \cos 2x$ in the interval $0 \le x \le 2p$.

A. $\frac{5p}{2}$

B. $\frac{5p}{3}$

C. $\frac{5p}{4}$

D. p

E. None of the above

21. $\tan\left(\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{4}\right)=?$

A. $3/5$

B. $5/3$

C. $3/4$

D. $4/3$

E. None of the above

22. Find the value of $\sin 20^\circ + \sin 40^\circ - \sin 80^\circ$

A. $1/2$

B. 1

C. $-1/2$

D. $\sqrt{3}/2$

E. 0

23.(**Tie Break No. 1**) Three circles each with a radius 8 are all tangent to each other. Find the area of the region between the circles

A. $64\sqrt{3} - 32p$

B. $16\sqrt{3} - 32p$

C. $256\sqrt{3} - 32p$

D. $32p - 64\sqrt{3}$

E. None of the above

24. $\dfrac{\sec 180^\circ \cot 45^\circ + \sin 30^\circ}{\cos 60^\circ + \cot^2 30^\circ} = ?$

A. $\dfrac{1}{7}$

B. $\dfrac{\sqrt{3}}{2}$

C. $-\dfrac{1}{7}$

D. $-\dfrac{\sqrt{3}}{2}$

E. None of the above

25. $\left[2\left(\cos 30^\circ + i \sin 30^\circ\right)\right]^3 = ?$

A. $8\sqrt{3} + 8i$

B. $\frac{3\sqrt{3}}{8} + \frac{1}{8}i$

C. $3\sqrt{3} - i$

D. $8i$

E. None of the above

26. If $\tan\left(45^\circ + x\right) = 2$, find the value of $\cos 2x + \sin 2x$

A. $-7/5$

B. $1/5$

C. $7/5$

D. $-1/5$

E. 1

27. How many positive 3-digit integers less than 500 can be formed using only the digits 1, 3, 5, and 7 if repetition of digits is allowed?

A. 32

B. 144

C. 400

D. 64

E. None of the above

28.(**Tie Break No. 2**) Given that *f* is a function such that $f(x+y)=f(x)+f(y)+x^2y+xy^2$ for all real values of *x* and *y*; and that $\lim_{x\to 0}\frac{f(x)}{x}=1$, then $f'(x)=?$

A. x

B. $2xy$

C. $4(x+y)$

D. x^2+1

E. x^2+2x+1

29. In the following 3×3 matrix, select 3 different entries at random. What is the probability a random selection of 3 entries has at least two in a same row or in a same column?

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

A. $1/14$

B. $3/7$

C. $14/27$

D. $11/14$

E. $13/14$

30. Find the exact value of $\sqrt{3+2\sqrt{2}}-\sqrt{3-2\sqrt{2}}$.

A. $-\sqrt{2}$

B. -2

C. $4\sqrt{2}$

D. $\sqrt{2}$

E. 2

31. If $r^2 + \frac{1}{r^2} = 4,$ Find the value of $\left(r + \frac{1}{r}\right)^4$

A. 24

B. 23

C. 16

D. 36

E. 38

32. A rectangular field is half as wide as it is long and is completely enclosed by x yards of fencing. What is the area of the rectangle in terms of x?

A. $\frac{x^2}{18}$

B. $2x^2$

C. $\frac{2x^2}{9}$

D. $\frac{x^2}{2}$

E. $\frac{x^2}{72}$

33.(**Tie Break No.3**) Solve $| x - 3 | + | x + 2 | < 11.$

A. $-2 < x < 3$

B. $-10 < x < 12$

C. $-5 < x < 6$

D. $x < 6$

E. $x > -5$

34. Find the domain of the real function $f(x) = \ln \ln \ln \ln x$

A. $\{x \mid x > e\}$

B. $\{x \mid x > 0\}$

C. $\{x \mid x > 1\}$

D. $\{x \mid x > e^e\}$

E. $\{x \mid x > e^2\}$

35. In order to receive an A in a college course it is necessary to obtain an average of 90% on five one-hour exams of 100 points each and on one final exam of 250 points. If a student scores 75, 82, 90, 91, and 92 on the one-hour exams, what is the minimum score (%) on the final exam that the person can receive and still earn an A?

A. 94%

B. 96%

C. 98%

D. 100%

E. Not possible

36. Let $\begin{vmatrix} 3-x & -6 \\ 5x-6 & 8 \end{vmatrix} = 36$. Solve for x

A. $\frac{9}{11}$

B. $\frac{24}{11}$

C. $\frac{12}{19}$

D. $\frac{9}{19}$

E. $\frac{24}{19}$

37. Solve the equation $\left(\sqrt{5}\right)^{x+4} = 25^{x}$

A. $x = 0$

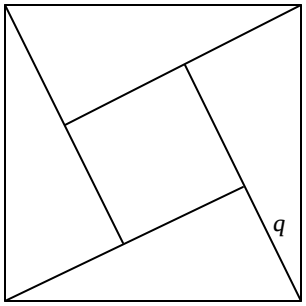
B. $x = \frac{4}{3}$

C. $x = \frac{8}{3}$

D. $x = 4$

E. $x = 0$ or $x = \frac{4}{3}$

38. In the given figure, q is the smaller acute angle in the four congruent right triangles. The area of the larger square is 25 and the area of the smaller square is 1. Find $\sin 2q$.



A. $22/25$

B. $13/25$

C. $17/25$

D. $24/25$

E. $12/25$

39. If $f(x) = x + 1$ then $f(1) - f(2) + f(3) - f(4) + ... + f(99) = ?$

A. 51

B. -149

C. -51

D. 100

E. None of above

40. The future value of a particular investment is calculated by the formula $F = P\left(1+\frac{r}{n}\right)^{nt}$, where P is the principal, r is the annual rate of return, n is the number of compounds per year, and t is the number of years. Find the formula for time to triple your investment.

A. $\dfrac{n\log 3}{1+\frac{r}{n}}$

B. $\dfrac{\log 3}{n\log\left(1+\frac{r}{n}\right)}$

C. $\log\dfrac{3}{1+\frac{r}{n}}$

D. $\dfrac{3n}{1+\frac{r}{n}}$

E. $\dfrac{3\log\left(1+\frac{r}{n}\right)}{n}$

41.(**Tie Break No. 4**) Examine the following functions. Which of them is neither even nor odd?

I. $f(x)=\log\left(x+\sqrt{x^2+1}\right)$ II. $f(x)=\ln\dfrac{1-x}{1+x}$,

III. $f(x)=\dfrac{x^2+x^3}{1+x}$ IV. $f(x)=x\left(\dfrac{1}{2^{-x}-1}+\dfrac{1}{2}\right)$

A. I

B. II

C. III

D. IV

E. None of the above

42. The solution set of the following system of inequalities is a triangle in the rectangular coordinate system. Find the range of the variable a.

$$\begin{cases} x - y \geq 0 \\ 2x + y \leq 2 \\ y \geq 0 \\ x + y \leq a \end{cases}$$

A. $a \geq \frac{4}{3}$

B. $0 < a \leq \frac{4}{3}$

C. $1 \leq a \leq \frac{4}{3}$

D. $0 < a \leq 1$ or $a \geq \frac{4}{3}$

E. $a \leq 1$ or $a \geq \frac{4}{3}$

43. If $f_1(x) = \frac{1}{2-x}$, $f_2 = f_1 \circ f_1$, $f_3 = f_1 \circ f_2$, $f_4 = f_1 \circ f_3$, …., find $f_{101}(3)$

A. -1

B. $\frac{199}{201}$

C. $\frac{99}{101}$

D. $\frac{199}{200}$

E. $\frac{99}{100}$

44. In the given figure, *A*, *B*, *C*, and *D* are four points on a circle. The segments *AC* and *BD* are extended and intersect at *E*. If $BD = 11$, $DE = 3$, and $AC = 19$, find the length of *CE*.

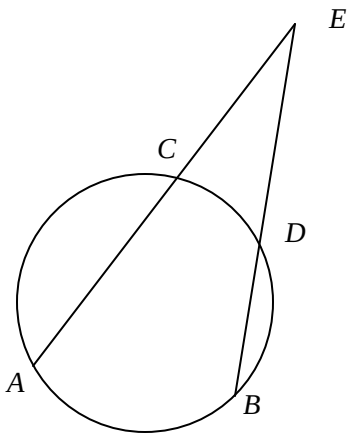
A. 5

B. 3

C. 4

D. 1

E. 2



45.(**Tie Break No. 5**) Find the minimum distance from points of $y = -x^2$ to the line $4x + 3y - 8 = 0$.

A. $8/3$

B. $4/3$

C. $4/5$

D. $8/15$

E. 3