

Ciphering

1. If $f(x) = x^2 + |x+1|$, then $f(-2) = ?$

$$f(-2) = (-2)^2 + |-2+1| = 4+1 = 5$$

Correct Answer: 5

2. What is the numerical value of $\sum_{n=1}^{\infty} \frac{2^{n+2}}{3^n}$?

$$\sum_{n=1}^{\infty} \frac{2^{n+2}}{3^n} = \sum_{n=1}^{\infty} \frac{8}{3} \left(\frac{2}{3}\right)^{n-1} = \frac{8}{3} \cdot \frac{1}{1-\frac{2}{3}} = \frac{8}{3} \cdot \frac{1}{\frac{1}{3}} = 8$$

Correct Answer: 8

3. You are given $P(A \cup B) = 0.7$ and $P(A \cup B') = 0.9$. Determine $P(A)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 \quad (1)$$

$$P(A \cup B') = P(A) + P(B') - P(A \cap B') = 0.9 \quad (2)$$

Add (1) and (2)

$$2P(A) + P(B) + P(B') - P(A \cap B) - P(A \cap B') = 1.6$$

$$2P(A) + 1 - P(A) = 1.6$$

$$P(A) = 0.6$$

Correct Answer: $P(A) = 0.6$

4. What is the area of a square whose diagonal is one unit longer than the length of its side?

Let x be the length of its side. $x^2 + x^2 = (x+1)^2$

$$x^2 - 2x - 1 = 0, x = 1 + \sqrt{2}, x^2 = (1 + \sqrt{2})^2 = 3 + 2\sqrt{2}$$

Correct Answer: $3 + 2\sqrt{2}$

5. Find the sum of the series $\frac{1}{3} - \frac{1}{2} + \frac{1}{9} - \frac{1}{4} + \frac{1}{27} - \frac{1}{8} + \dots$

$$\begin{aligned} \frac{1}{3} - \frac{1}{2} + \frac{1}{9} - \frac{1}{4} + \frac{1}{27} - \frac{1}{8} + \dots &= \left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \right) - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right) \\ &= \frac{1}{3} \cdot \frac{1}{1 - \frac{1}{3}} - \frac{1}{2} \cdot \frac{1}{1 - \frac{1}{2}} = \frac{1}{2} - 1 = -\frac{1}{2} \end{aligned}$$

Correct Answer: $-\frac{1}{2}$

6. If $\sin\left(\frac{\pi}{2} + \theta\right) = \frac{3}{5}$, find $\cos 2\theta$.

$$\sin\left(\frac{\pi}{2} + \theta\right) = \cos \theta = \frac{3}{5} \text{ and } \cos 2\theta = 2\cos^2 \theta - 1 = 2\left(\frac{3}{5}\right)^2 - 1 = -\frac{7}{25}$$

Correct Answer: $-\frac{7}{25}$

7. A piece of coaxial cable $11\frac{2}{3}$ feet long is to be divided into two parts such that one part is $\frac{2}{3}$ the length of the other. What is the length of the shorter piece?

Let x be the length of the shorter piece.

$$x + \frac{3}{2}x = \frac{35}{3}, \quad \frac{5}{2}x = \frac{35}{3}, \quad x = \frac{14}{3} \text{ or } x = 4\frac{2}{3}$$

Correct Answer: $4\frac{2}{3}$ feet

8. Solve for x : $9^{2x} \cdot 27^{x^2} = 3^{-1}$

$$\begin{aligned} (3^2)^{2x} \cdot (3^3)^{x^2} &= 3^{-1}, \quad 3^{4x} \cdot 3^{3x^2} = 3^{-1}, \quad 3^{3x^2+4x} = 3^{-1}, \quad 3x^2 + 4x + 1 = 0, \\ (3x+1)(x+1) &= 0, \quad x = -1 \text{ or } x = -\frac{1}{3} \end{aligned}$$

Correct Answer: $x = -1$ or $x = -\frac{1}{3}$

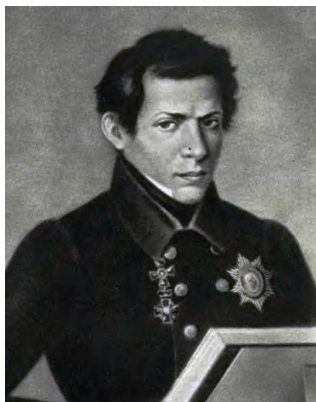
9. The following table gives values of f , f' , g , and g' at selected values of x . If $h(x) = f(g(x))$, then $h'(1) = ?$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	6	5	3	-2
1	3	-3	-1	2
3	1	-2	2	3

$$h'(1) = f'(g(1)) \cdot g'(1) = f'(-1) \cdot 2 = 5 \cdot 2 = 10$$

Correct Answer: 10

10. Who am I?



Hint: I developed a non-Euclidean geometry.

Correct Answer: Nikolai Lobachevsky

11. If the length of a rectangle is increased by 30% while its width is decreased by 20%, give the percent of increase/decrease in the area.

Let l be the length of the rectangle and w the width. The ratio of the area change is $\frac{(1.3l) \cdot (0.8w)}{lw} = 1.04$. The area increases 4%.

Correct Answer: 4%

12. The seventh term of an arithmetic sequence is 41 and the thirteenth term is 77. What is the twentieth term in this arithmetic sequence?

$$\begin{cases} a_7 = a_1 + 6d = 41 \\ a_{13} = a_1 + 12d = 77 \end{cases}$$

$$d = 6 \text{ and } a_1 = 5$$

$$a_{20} = 5 + 19 \cdot 6 = 119$$

Correct Answer: 119

13. $\sin\left(2\cos^{-1}\frac{1}{\sqrt{5}}\right) = ?$

Let $\theta = \cos^{-1}\frac{1}{\sqrt{5}}$. Then $\cos\theta = \frac{1}{\sqrt{5}}$ and $\sin\theta = \frac{2}{\sqrt{5}}$.

$$\sin\left(2\cos^{-1}\frac{1}{\sqrt{5}}\right) = \sin 2\theta = 2\sin\theta\cos\theta = 2 \cdot \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} = \frac{4}{5}$$

Correct Answer: $\frac{4}{5}$

14. $(1+i)^{12} = ?$

$$(1+i)^{12} = \left((1+i)^2\right)^6 = (2i)^6 = 2^6 i^6 = -64$$

Correct Answer: -64

15. Al can complete a piece of work in 10 days. After he has worked 2 days, Bill comes to help him and together they complete the work in 3 days. In how many days could Bill have completed the original job working alone?

Let x be the days needed for Bill to complete the original job working alone.

$$2 \cdot \frac{1}{10} + 3\left(\frac{1}{10} + \frac{1}{x}\right) = 1, \quad \frac{1}{2} + \frac{3}{x} = 1, \quad \frac{3}{x} = \frac{1}{2}, \quad \text{and } x = 6.$$

Correct Answer: 6 days

16. Find the maximum value of the trigonometric function $y = \sin\theta + \cos\theta$.

$$y = \sin\theta + \cos\theta = \sqrt{2}\left(\cos\frac{\pi}{4}\sin\theta + \sin\frac{\pi}{4}\cos\theta\right) = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) \leq \sqrt{2}$$

Correct Answer: $\sqrt{2}$

17. Determine the ninth term in descending order in the expansion of $\left(x - \frac{1}{\sqrt{x}}\right)^{12}$.

$$\text{The ninth term is } \binom{12}{8} x^{12-8} \left(-\frac{1}{\sqrt{x}}\right)^8 = \frac{12!}{8! \cdot 4!} \cdot x^4 \cdot \frac{1}{x^4} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} = 495$$

Correct Answer: 495

18. Simplify $\frac{2^{n+4} - 2(2^n)}{2(2^{n+3})}$

$$\frac{2^{n+4} - 2(2^n)}{2(2^{n+3})} = \frac{2^n(2^4 - 2)}{2^n \cdot 2(2^3)} = \frac{16 - 2}{16} = \frac{7}{8}$$

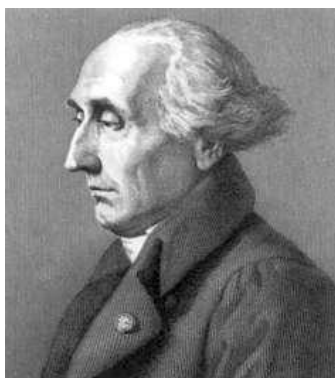
Correct Answer: $\frac{7}{8}$

19. Determine the value of the expression $\log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ca}\right) + \log\left(\frac{c^2}{ab}\right)$

$$\log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ca}\right) + \log\left(\frac{c^2}{ab}\right) = \log\left(\frac{a^2 b^2 c^2}{bccaab}\right) = \log 1 = 0$$

Correct Answer: 0

20. Who am I?



Hint: The multiplier method in mathematical optimization is named after me.

Correct Answer: Joseph Louis Lagrange