1. For what value of n does ${}_{n+1}P_3 = {}_nP_4$?

${}_{n+1}P_3 = {}_nP_4$ implies $(n+1)n(n-1) = n(n-1)(n-2)(n-3)$.
$n+1 = (n-2)(n-3)$
$n^2 - 6n + 5 = 0$, $(n-5)(n-1) = 0$
We have to have $n \geq 4$. Therefore, $n = 5$

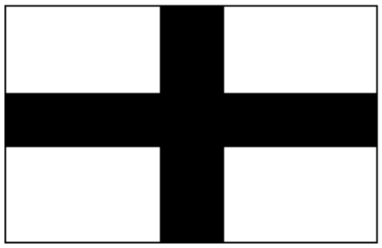
Correct Answer: 5

2. A box contains 5 red and 4 white marbles. Two marbles are drawn successively from the box without replacement. If the second marble drawn is white, what is the probability that the first one drawn was also white?

There are $8 \cdot 4$ ways with the second marble drawn white and $4 \cdot 3$ ways with the both marbles white. If the second marble drawn is white, the probability that the first one drawn was also white is $\frac{4 \cdot 3}{8 \cdot 4} = \frac{3}{8}$

Correct Answer: $\frac{3}{8}$

3. A flag is to be made with a black cross (all four arms the same thickness) centered on a 3 meter by 4 meter white rectangle as shown in the figure. How wide should the arms of the cross be so that exactly half of the flag's area is white?



Let x be the width of the arms. $x < 3$
$(3-x)(4-x) = \frac{1}{2}(3)(4)$
$x^2 - 7x + 6 = 0$
$x = 1$

Correct Answer: 1 meter

4. $(1+i)^4 + (1-i)^4 = ?$

$\left((1+i)^2\right)^2 + \left((1-i)^2\right)^2 = (2i)^2 + (-2i)^2 = -4 + (-4) = -8$

Correct Answer: -8

5. If θ is an acute angle and $\sin\theta = \frac{3}{5}$, find the exact value of $\tan 2\theta$.

$$\tan\theta = \frac{3}{4}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{24}{7}$$

Correct Answer: $\frac{24}{7}$

6. $\log_3 8 \cdot \log_8 9 = ?$

$$\log_3 8 \cdot \log_8 9 = \frac{\log 8}{\log 3} \cdot \frac{\log 9}{\log 8} = \frac{2\log 3}{\log 3} = 2$$

Correct Answer: 2

7. In 2009 the population of Abra increased by 20% while the population of Cadabra decreased by 10%, after which the two populations were equal. What percent of the original population of Cadabra was the original population of Abra?

Let *a* be the original population of Abra and *c* the original population of Cadabra.

$1.2a = 0.9c$, $\frac{a}{c} = \frac{0.9}{1.2} = 0.75 = 75\%$

Correct Answer: 75%

8. A package of a dozen eggs contains two bad ones. In how many different ways can you select 3 of the 12 eggs so that none of the bad ones are included?

$${}_{10}C_3 = \frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1} = 120$$

Correct Answer: 120

9. If the rectangle were 2 units wider and 3 units longer, it would be 64 square units larger. If it were 3 units wider and 2 units longer, it would be 68 square units larger. What are the dimensions of the original rectangle?

Let x be the width and y length of the rectangle.
$(x+2)(y+3) = xy+64,\ 3x+2y=58$
$(x+3)(y+2) = xy+68,\ 2x+3y=62$
$x-y=-4,\ 3(y-4)+2y=58,\ 5y=70$, and $y=14$
$x=10$

Correct Answer: 10 by 14

10. Who am I?

Hint: Trigonometric series is after my name.

Correct Answer: Jean Joseph Fourier

11. Given $f(x) = -1+3(x-1)-3(x-1)^2+(x-1)^3$, find $f(8)$

$f(x) = -1+3(x-1)-3(x-1)^2+(x-1)^3$
$= ((x-1)-1)^3 = (x-2)^3$
$f(8) = 6^3 = 216$

Correct Answer: 216

12. Let $\{a_n\}$ be an arithmetic sequence. if $a_1 + a_3 + a_5 = 105$ and $a_2 + a_4 + a_6 = 99$, find a_{20}

Let *d* be the common difference of the sequence.

$a_1 + a_3 + a_5 = 3a_1 + 6d = 105$

$a_2 + a_4 + a_6 = 3a_1 + 9d = 99$

$d = -2,\ a_1 = 39$

$a_{20} = a_1 + 19d = 39 + 19(-2) = 1$

Correct Answer: 1

13. The average of three numbers is 55. The second is 1 more than twice the first, and the third is 4 less than three times the first. What is the largest number?

Let *x, y,* and *z* be the numbers.

$x + y + z = 3(55) = 165,\ y = 1 + 2x,\ z = 3x - 4$

$x + (1 + 2x) + (3x - 4) = 165,\ 6x = 168,\ x = 28,\ y = 57$, and $z = 80$

Correct Answer: 80

14. A man bought some plates, ⅔ of them were cracked; ½ of them were chipped; and ¼ were both chipped and cracked. All but 2 were either chipped or cracked. How many did he buy?

Let *x* be number of the plates the man bought.

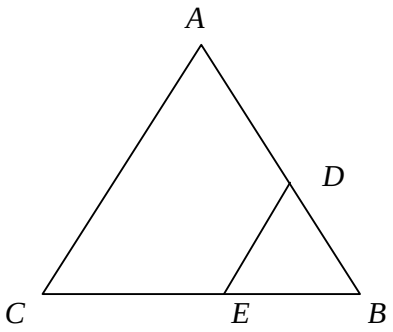
$x - \left(\frac{2}{3}x + \frac{1}{2}x\right) + \frac{1}{4}x = 2,\ \frac{1}{12}x = 2$, and $x = 24$

Correct Answer: 24

15. A triangular corner with sides *DB* and *EB* each of length 1 is cut from an equilateral triangle ABC with sides of length 3. What is the perimeter of the remaining quadrilateral?

$DE // AC,\ DE = DB = EB = 1,\ AD = CE = 2$

$AC + CE + ED + AD = 3 + 2 + 1 + 2 = 8$



Correct Answer: 8

16. $f(x) = \log_a \frac{1-x}{1+x}$ and $f(b) = 5$. Find $f(-b)$.

$$f(-b) = \log_a \frac{1+b}{1-b} = -\log_a \frac{1-b}{1+b} = -f(b) = -5$$

Correct Answer: -5

17. If $f(x+1) = \frac{x}{x+1}$, then $f(x-1) = ?$ Simplify your result.

$$f(x-1) = f((x-2)+1) = \frac{(x-2)}{(x-2)+1} = \frac{x-2}{x-1}$$

Correct Answer: $\frac{x-2}{x-1}$ or $1 - \frac{1}{x-1}$

18. Find x so that x, $x+2$, and $x+3$ are consecutive terms of a geometric sequence.

$(x+2)^2 = x(x+3)$, $x^2 + 4x + 4 = x^2 + 3x$, $x = -4$

Correct Answer: -4

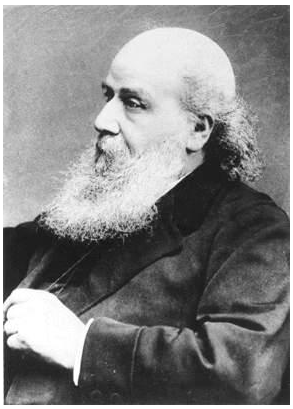
19. Find the domain of the function $f(x) = \sqrt{\log_{1/2}(4x-3)}$. Write your answer in interval notation.

$\log_{1/2}(4x-3) \geq 0$, $0 < 4x - 3 \leq 1$, $3 < 4x \leq 4$, $\frac{3}{4} < x \leq 1$

The domain is $\left(\frac{3}{4}, 1\right]$

Correct Answer: $\left(\frac{3}{4}, 1\right]$

20. Who am I?



Hint: We have the same family name.

Correct Answer: James Joseph Sylvester