1. $\dfrac{2\sqrt{2}-i^3}{1-2i\sqrt{2}}=?$

$$\frac{2\sqrt{2}-i^3}{1-2i\sqrt{2}}=\frac{2\sqrt{2}+i}{1-2i\sqrt{2}}=\frac{(2\sqrt{2}+i)(1+2i\sqrt{2})}{(1-2i\sqrt{2})(1+2i\sqrt{2})}=\frac{2\sqrt{2}+8i+i+2i^2\sqrt{2}}{1-(2i\sqrt{2})^2}$$
$$=\frac{2\sqrt{2}+9i-2\sqrt{2}}{1+8}=\frac{9i}{9}=i$$

Correct answer: D

2. How many pairs of natural numbers (m,n) satisfy the equation $\dfrac{4}{m}+\dfrac{2}{n}=1$?

We have to have $m\ge 5$ and $n\ge 3$

$\dfrac{2}{n}\le\dfrac{2}{3}$ and $\dfrac{4}{m}\ge 1-\dfrac{2}{3}=\dfrac{1}{3}$, then $m\le 12$; $\dfrac{4}{m}\le\dfrac{4}{5}$ and $\dfrac{2}{n}\ge 1-\dfrac{4}{5}=\dfrac{1}{5}$, then $n\le 10$

Trying these numbers, we have $\dfrac{4}{12}+\dfrac{2}{3}=1$, $\dfrac{4}{8}+\dfrac{2}{4}=1$, $\dfrac{4}{6}+\dfrac{2}{6}=1$, and $\dfrac{4}{5}+\dfrac{2}{10}=1$

Correct answer: D

3. **(Tie Break No. 1)** If the surface area of a closed rectangular box is 11 and the sum of the lengths of its 12 edges is 24. What is the length of its diagonal?

Let *x*, *y*, and *z* be the lengths of the edges of the rectangular box

$$2xy+2yz+2zx=11$$
$$4x+4y+4z=24$$
$$x+y+z=6$$
$$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx=36$$
$$x^2+y^2+z^2=36-11=25$$
$$\sqrt{x^2+y^2+z^2}=5$$

Correct answer: C

4. The line l passes through the point $(-2,0)$ and intersects the circle $x^2+y^2=2x$ at two points. The range of the slope m of the line l is

The equation of the line passing through the point $(-2,0)$ is $y=m(x+2)$. Substituting into the equation of the circle, we have

$$x^2+(m(x+2))^2=2x$$
$$x^2+m^2x^2+4m^2x+4m^2=2x$$
$$(m^2+1)x^2+(4m^2-2)x+4m^2=0$$

When $(4m^2-2)^2-4(m^2+1)(4m^2)>0$, the equation has two solutions.

$$16m^4-16m^2+4-16m^4-16m^2>0$$
$$2-16m^2>0$$
$$-\frac{\sqrt{2}}{4}<m<\frac{\sqrt{2}}{4}$$

Correct answer: B

5. Find the period of the function $f(x)=\sin\frac{x}{2}+\cos\frac{x}{3}$.

The period of $y=\sin\frac{x}{2}$ is $\frac{2\pi}{\frac{1}{2}}=4\pi$ and the period of $y=\cos\frac{x}{3}$ is $\frac{2\pi}{\frac{1}{3}}=6\pi$. The least multiple of 4 and 6 is 12.

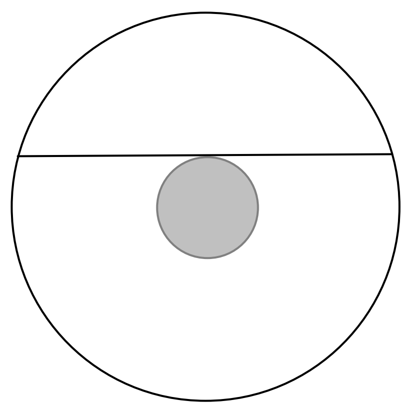
Correct answer: D

6. The circles in the figure are concentric. The chord is tangent to the inner circle and has length 14. What is the exact area of the non-shaded region?

Let R be the radius of large circle and r be the radius of small circle. Then the area of the non-shaded region is $\pi R^2-\pi r^2=\pi\left(R^2-r^2\right)$.

By the Pythagorean Theorem, $R^2-r^2=49$ since half the chord length is 7.

Therefore, the exact area of the non-shaded region is 49π .



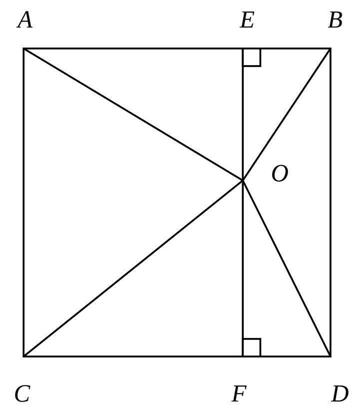
Correct answer: A

7. $\{a_k\}$ is an arithmetic sequence. $a_1 = 18$ and the common difference $d = -3$. Find all n such that the partial sum $S_n = \sum_{k=1}^{n} a_k$ has the maximum value.

The k^{th} term formula of the series is $a_k = a_1 + (k-1)d = 21 - 3k$
$a_7 = 0$ and $a_k < 0$ when $k \geq 8$.
$S_6 = S_7 = 18 + 15 + 12 + 9 + 6 + 3 = 63$ is the maximum value of S_n

Correct answer: A

8. **(Tie Break No. 2)** In the figure, the length of the sides of the square $ABCD$ is 8. $OA = 7$ and $OB = 3$. Find the area of the triangle ΔOCD



By Heron's formula the area of ΔOAB is
$\sqrt{9(9-8)(9-7)(9-3)} = 6\sqrt{3}$

The sum of the areas of ΔOAB and ΔOCD is equal to
$\frac{1}{2}AB \cdot OE + \frac{1}{2}CD \cdot OF = \frac{1}{2}AB \cdot FE = 32$, that is one half of area of the square.

The area of the ΔOCD is $32 - 6\sqrt{3}$

Correct answer: C

9. A circle passes though the points $(0, 0)$ and $(2, 4)$. Its center is on the line $2x - y = 5$. Find the equation of the circle.

Let (h, k) be the center of the circle. We have $(0-h)^2 + (0-k)^2 = (h-2)^2 + (k-4)^2$.
(h, k) is the solution set of the system.

$$\begin{cases} 4h + 8k = 20 \\ 2h - k = 5 \end{cases}$$

$(h, k) = (3, 1)$
Let the equation of the circle be $(x-3)^2 + (y-1)^2 = r^2$. The circle passes through $(0, 0)$.
$r^2 = (-3)^2 + (-1)^2 = 10$

Correct answer: B

10. For what values of a and b are the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \ (a > 0, b > 0)$ perpendicular?

The equations of two asymptotes are $y = \pm \frac{b}{a} x$

They are perpendicular to each other if and only if $\frac{b}{a}\left(-\frac{b}{a}\right) = -1$

$$a^2 = b^2$$
$$a = b$$

. Correct answer: B

11. A total of 120 numbers can be formed using all five digits 1, 2, 3, 4, and 5. If these numbers are arranged in increasing order (12345, 12354, 12435, 12453 up to 54321) which one is 75th in this order?

A total of $24 \times 3 = 72$ numbers begin with 1, 2 and 3.
The following numbers are 41235, 41253, 41325, $\cdots$ in increasing order.
The 75th number is 41325.

Correct answer: D

12. If the three sides a, b, and c of triangle ΔABC are also the first three consecutive terms of a geometric sequence and $c = 2a$, find $\cos B$

Let $b = ar$ and $c = ar^2$. We have $ar^2 = 2a$ and $r = \sqrt{2}$ Therefore, $b = a\sqrt{2}$ and $c = 2a$.

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{a^2 + (2a)^2 - \left(a\sqrt{2}\right)^2}{2a(2a)} = \frac{3}{4}$$

Correct answer: B

13. Solve $|x^2 - 2x - 16| = 8$.

$x^2 - 2x - 16 = 8$ or $x^2 - 2x - 16 = -8$
$x^2 - 2x - 24 = 0$ or $x^2 - 2x - 8 = 0$
$(x-6)(x+4) = 0$ or $(x-4)(x+2) = 0$
$x = 6$, $x = -4$, $x = 4$, or $x = -2$

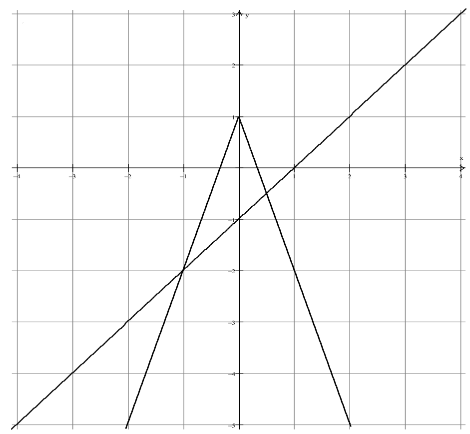
Correct answer: B

14. The area of the solution region of the system of the inequalities $\begin{cases} y \geq x-1 \\ y \leq -3|x|+1 \end{cases}$ is

As shown in the figure, the solution region is a triangle with vertices $(0,1)$, $(-1,-2)$, and $\left(\frac{1}{2}, -\frac{1}{2}\right)$

The area of the triangle is the absolute value of the determinant

$$\frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 1 & -1 & -2 \\ 1 & 1/2 & -1/2 \end{vmatrix} = \frac{3}{2}$$



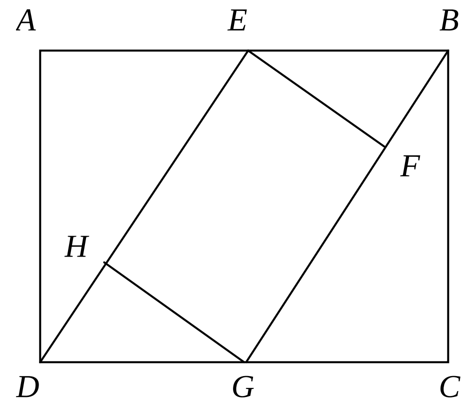
Correct answer: B

15. Find the real solutions of the equation $\sqrt{\log x - 3} = \log x - 3$.

Squaring the both sides, we have $\log x - 3 = (\log x)^2 - 6\log x + 9$.
$(\log x)^2 - 7\log x + 12 = 0$
$(\log x - 3)(\log x - 4) = 0$
$\log x = 3$ or $\log x = 4$
$x = 1{,}000$ or $x = 10{,}000$
Both of them are checked.

Correct answer: C

16. **(Tie Break No. 3)** In rectangle *ABCD*, $AB = CD = 4$ and $AD = CB = 3$. Points *E* and *G* are the midpoints of *AB* and *CD* respectively. $EF \perp GB$ and $GH \perp DE$. Find the area of rectangle *EFGH* shown in the figure.



$DE = \sqrt{(AD)^2 + (AE)^2} = \sqrt{3^2 + 2^2} = \sqrt{13}$

Since $\Delta ADE \sim \Delta HGD$, $\dfrac{DG}{DE} = \dfrac{GH}{AD} = \dfrac{DH}{AE}$

$GH = \dfrac{AD \cdot DG}{DE} = \dfrac{3 \cdot 2}{\sqrt{13}} = \dfrac{6}{\sqrt{13}}$

$DH = \dfrac{AE \cdot DG}{DE} = \dfrac{2 \cdot 2}{\sqrt{13}} = \dfrac{4}{\sqrt{13}}$

$EH = DE - DH = \sqrt{13} - \dfrac{4}{\sqrt{13}} = \dfrac{9}{\sqrt{13}}$

Area of rectangle *EFGH* is equal to $GH \cdot EH = \dfrac{54}{13}$

Correct answer: B

17. Find all values of *x* where the graph of $f(x) = \dfrac{3x^2 + 5x - 1}{x^2 - x + 3}$ intersects its horizontal asymptote.

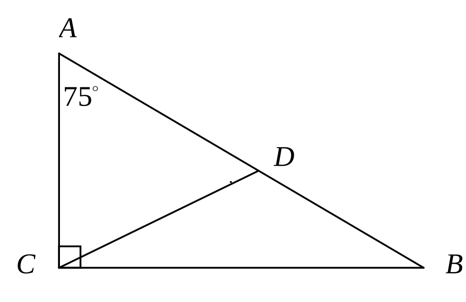
$f(x)$ has a horizontal asymptote $y = 3$. Solve the equation

$$\frac{3x^2 + 5x - 1}{x^2 - x + 3} = 3$$

$$3x^2 + 5x - 1 = 3x^2 - 3x + 9, \quad 8x = 10, \text{ and } \quad x = 5/4$$

Correct answer: A

18. ΔABC is a right triangle with $\angle A = 75^\circ$ and $\angle C = 90^\circ$. Point *D* is on side *AB*. If $AD = CD$ and $AC = 10$, find the area of ΔCBD.



The area of ΔACD is equal to $\dfrac{1}{2} AC\left(\dfrac{1}{2} AC \cdot \tan 75^\circ\right)$

The area of ΔABC is equal to $\dfrac{1}{2} AC\left(AC \cdot \tan 75^\circ\right)$

The area of ΔCBD is equal to

$\dfrac{1}{4}(AC)^2 \tan 75^\circ = \dfrac{1}{4}(10^2)\dfrac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$

$= 25\left(\dfrac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}}\right) = 25\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right) \cdot \dfrac{\sqrt{3}+1}{\sqrt{3}+1} = 25(2 + \sqrt{3})$

Correct answer: D

19. If $f(x) = x^2 - 2$ and $g(x) = x - 3$, find all values of x such that $(f \circ g)(x) = (g \circ f)(x)$

$$(f \circ g)(x) = f(g(x)) = (x-3)^2 - 2$$
$$(g \circ f)(x) = g(f(x)) = (x^2 - 2) - 3$$
$$(x-3)^2 - 2 = (x^2 - 2) - 3$$
$$x^2 - 6x + 9 - 2 = x^2 - 5$$
$$x = 2$$

Correct answer: C

20. Solve the equation $a^2 = \log_b a^3$ for a, given $b = a^5$ and a and b are real numbers.

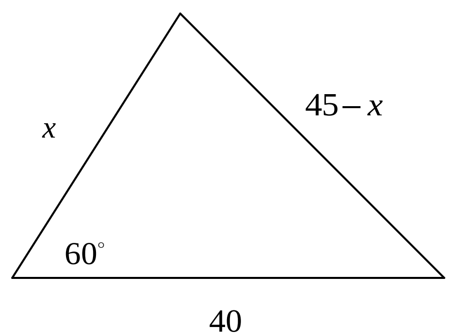
Using the base change formula, $\log_b a^3 = \dfrac{\log a^3}{\log b} = \dfrac{\log a^3}{\log a^5} = \dfrac{3\log a}{5\log a} = \dfrac{3}{5}$

$a^2 = \dfrac{3}{5}$ and $a = \sqrt{\dfrac{3}{5}} = \dfrac{\sqrt{15}}{5}$

Correct answer: C

21. The base of a triangle is 40 feet and one of the base angles is 60°. The sum of the other two sides is 45 feet. What is the measure of the shortest side of the triangle?

$$(45 - x)^2 = x^2 + 40^2 - 2(40)x\cos 60^\circ$$
$$2025 - 90x + x^2 = x^2 + 1600 - 40x$$
$$50x = 425$$
$$x = 8.5$$



Correct answer: D

22. A particular sphere and a particular right circular cylinder have the same volume. If the radius of the cylinder is 6 times its height, find the ratio of the surface area of the sphere to the surface area of the cylinder.

Let r_1 be the radius of the sphere and r_2 the radius of the cylinder. The volume of the sphere is $\frac{4}{3}\pi r_1^3$ and the volume of the cylinder is $\pi r_2^2 h = \pi r_2^2 \frac{1}{6} r_2 = \frac{1}{6}\pi r_2^3$

$$\frac{4}{3}\pi r_1^3 = \frac{1}{6}\pi r_2^3 \text{ and } r_2 = 2r_1$$

The surface of the sphere is $4\pi r_1^2$ and the surface of the cylinder is

$$2\pi r_2^2 + 2\pi r_2 \cdot \frac{1}{6} r_2 = 2\pi (2r_1)^2 + \frac{1}{3}\pi (2r_1)^2 = \frac{28}{3}\pi r_1^2$$

The ratio of the surface area of the sphere to the surface area of the cylinder is $\frac{4}{\frac{28}{3}} = \frac{3}{7}$.

Correct answer: B

23. Find the domain of the function $f(x) = 1 + \dfrac{1}{1 + \dfrac{1}{\ln x}}$.

The domain of the function is $\left\{ x \mid x > 0,\ \ln x \neq 0,\ 1 + \frac{1}{\ln x} \neq 0 \right\}$

$\ln x = 0$, $x = 1$

$1 + \frac{1}{\ln x} = 0$, $\ln x = -1$, $x = \frac{1}{e}$

The domain of the function is $\left(0, \frac{1}{e}\right) \cup \left(\frac{1}{e}, 1\right) \cup (1, \infty)$ in interval notation.

Correct answer: A

24. A hiker starts at noon at the top of a mountain 2 miles high, and walks down a straight path to the bottom at a rate of 5 miles per hour. Another hiker starts at noon at the bottom of same mountain and walks up the same path at a rate of 3 miles per hour. The straight path is 16 miles long. What altitude are the hikers at when they meet?

The slope of the mountain is $2/16 = 1/8$. Let x be the distance from the bottom to the point where they meet. We have

$$\frac{x}{3} = \frac{16 - x}{5},\quad 5x = 48 - 3x,\quad x = 6$$

The altitude of the meeting point is $6 \cdot \frac{1}{8} = \frac{3}{4}$

Correct answer: E

25. Solve the inequality $\frac{10}{x^2 - x - 12} > -1$

The inequality is equivalent to $1 + \frac{10}{x^2 - x - 12} > 0$ or $\frac{x^2 - x - 2}{x^2 - x - 12} > 0$

Let $f(x) = \frac{x^2 - x - 2}{x^2 - x - 12} = \frac{(x-2)(x+1)}{(x-4)(x+3)}$

The zeros of the denominator and numerator $\{-3, -1, 2, 4\}$ divide the number line into 5 intervals. $f(-4) > 0$, $f(-2) < 0$, $f(0) > 0$, $f(3) < 0$, and $f(5) > 0$. By the continuity of $f(x)$ on these intervals, the solution set of the inequality is $(-\infty, -3) \cup (-1, 2) \cup (4, \infty)$.

Correct answer: D

26. Suppose that a rope surrounds the earth at the equator. The rope is lengthened by 10 *ft*. By how much is the rope raised above the earth?

Let *R* be the radius of the earth *h* the height raised above the earth after the rope is lengthened.

$$2\pi R + 10 = 2\pi(R + h)$$

$$10 = 2\pi h \text{ and } h = \frac{5}{\pi}$$

Correct answer: C

27. **(Tie Break No. 4)** A man is digging a hole and standing in it. He is 5 *ft* 10 *in* tall. He says that he is one fourth done. When he is finished, the top of his head will be 3 times as far below the ground as it is now above ground. How deep will the hole be when finished?

The man is 70 inches tall. Let *d* be the depth of the hole when finished. The top of his head is $\left(70 - \frac{1}{4}d\right)$ above the ground now. We have

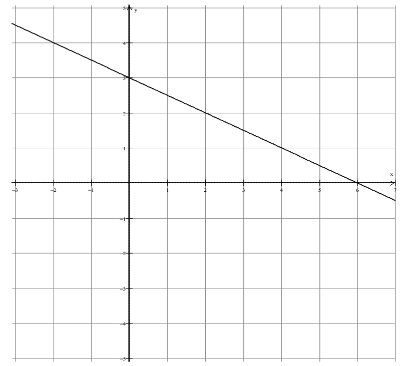
$$d = 3\left(70 - \frac{1}{4}d\right) + 70$$

$$\frac{7}{4}d = 280$$

$$d = 160\,in = 13\,ft\ 4\,in$$

Correct answer: B

28. Maximize $P(x, y) = 5x + 9y$ subject to $x \geq 0,\ \ y \geq 0$, and $x + 2y \leq 6$.



The solution set of $x \geq 0,\ \ y \geq 0$, and $x + 2y \leq 6$ is a triangle with vertices $(0,0), (0,3),$ and $(6,0)$.
$P(0,0) = 0$, $P(0,3) = 27$, and $P(6,0) = 30$
The maximum value of $P(x, y)$ is 30.

Correct answer: C

29. After an initial deposit of x dollars, the amount of money in a certain fund is doubled at the end of each month for 5 months. If at the end of the 5-month period there is a total of \$560 in the fund, how much money was in the fund at the beginning of the third month?

$$x \cdot 2^5 = 560$$
$$x = 17.5$$
$$17.5(2^2) = 70$$

Correct answer: C

30. In the expression $\frac{x}{2} + \frac{2}{x} + \frac{x}{2} + \frac{2}{x} + \ldots$ if each odd-numbered term is $\frac{x}{2}$ and each even-numbered term is $\frac{2}{x}$, then what is the sum of the first 56 terms of the expression?

The sum of the odd-numbered terms of the first 56 terms is $28\left(\frac{x}{2}\right) = 14x$ and the sum of all the even-numbered terms of the first 56 terms is $28\left(\frac{2}{x}\right) = \frac{56}{x}$.

$$14x + \frac{56}{x} = \frac{14x^2 + 56}{x}$$

Correct answer: E

31. If $2 \le x \le 4$ and $2 \le y \le 4$, what is the maximum possible value of $x - \frac{x}{y}$?

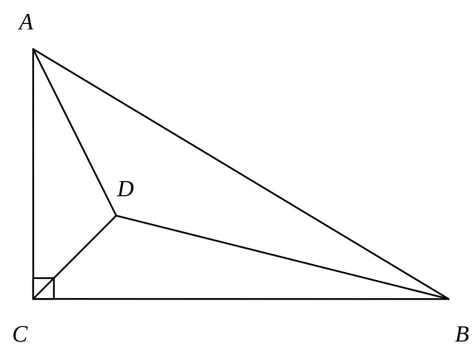
$$x - \frac{x}{y} = x\left(1 - \frac{1}{y}\right)$$

When $x = 4$ and $y = 4$, the two factors have the maximum values, respectively and so does the product.

$$4\left(1 - \frac{1}{4}\right) = 3$$

Correct answer: D

32. In the figure, the right triangle *ABC* has $\angle C = 90^\circ$ and contains a point *D*. If $AD = 10$, $CD = 6$, and $\angle ADC = \angle CDB = \angle BDA$. Find BD.



$\angle ADC = \angle CDB = \angle BDA = 120^\circ$

$(AC)^2 = (AD)^2 + (CD)^2 - 2(AD)(CD)\cos 120^\circ$

$= 10^2 + 6^2 - 2(10)(6)(-0.5) = 196$ (1)

$(BC)^2 = (CD)^2 + (BD)^2 - 2(CD)(BD)\cos 120^\circ$

$= 6^2 + (BD)^2 - 2(6)(BD)(-0.5)$ (2)

$(AB)^2 = (AD)^2 + (BD)^2 - 2(AD)(BD)\cos 120^\circ$

$= 10^2 + (BD)^2 - 2(10)(BD)(-0.5)$ (3)

$(AC)^2 + (BC)^2 = (AB)^2$ (4)

Substituting (1), (2), (3), $AD = 10$, and $CD = 6$ into (4), we have

$196 + 36 + (BD)^2 + 6(BD) = 100 + (BD)^2 + 10(BD)$

Solving the equation, we have $BD = 33$

Correct answer: D

33. Given a rectangle, if one side is decreased by 3 and the adjacent side is increased by 2, it forms a square with area 25. What is the perimeter of the rectangle?

Let *x* and *y* be the lengths of two adjacent sides of the rectangle.

$$x - 3 = y + 2 = 5$$

$$x = 8 \text{ and } y = 3$$

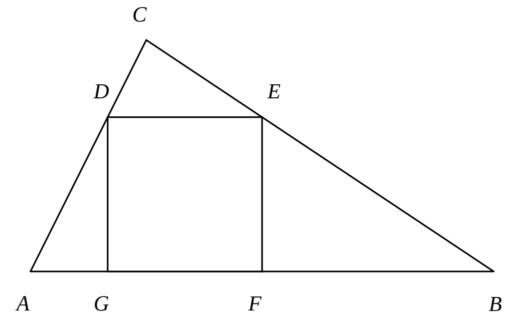
$$2x + 2y = 2(8) + 2(3) = 22$$

Correct answer: D

34. In the figure, the right triangle ABC has $\angle C = 90^\circ$, $BC = a$, and $AC = b$. The square $DEFG$ is inscribed in the triangle. Find the perimeter of the square.

$AB = \sqrt{(BC)^2 + (AC)^2} = \sqrt{a^2 + b^2}$

Let x be the length of the sides of the square



$\Delta DCE \sim \Delta ACB$, $\dfrac{CE}{DE} = \dfrac{CB}{AB}$, $CE = x\left(\dfrac{a}{\sqrt{a^2+b^2}}\right)$

$\Delta EFB \sim \Delta ACB$, $\dfrac{EB}{EF} = \dfrac{AB}{AC}$, $EB = x\left(\dfrac{\sqrt{a^2+b^2}}{b}\right)$

$a = CB = CE + EB = x\left(\dfrac{a}{\sqrt{a^2+b^2}} + \dfrac{\sqrt{a^2+b^2}}{b}\right)$

$a = x\left(\dfrac{a^2+ab+b^2}{b\sqrt{a^2+b^2}}\right)$, $x = \dfrac{ab\sqrt{a^2+b^2}}{a^2+ab+b^2}$, and $4x = \dfrac{4ab\sqrt{a^2+b^2}}{a^2+ab+b^2}$

Correct answer: C

35. Find $\lim\limits_{n\to\infty} \dfrac{\left(1+\frac{1}{2}+\frac{1}{2^2}+\cdots+\frac{1}{2^n}\right)2^n}{1+3\cdot 2^n}$.

$$\lim_{n\to\infty} \frac{\left(1+\frac{1}{2}+\frac{1}{2^2}+\cdots+\frac{1}{2^n}\right)2^n}{1+3\cdot 2^n} = \lim_{n\to\infty} \frac{\left(\dfrac{1-\frac{1}{2^{n+1}}}{1-\frac{1}{2}}\right)\cdot 2^n}{1+3\cdot 2^n} = \lim_{n\to\infty} \frac{\dfrac{1-\frac{1}{2^{n+1}}}{1-\frac{1}{2}}}{\frac{1}{2^n}+3} = \frac{\dfrac{1}{1-\frac{1}{2}}}{3} = \frac{2}{3}$$

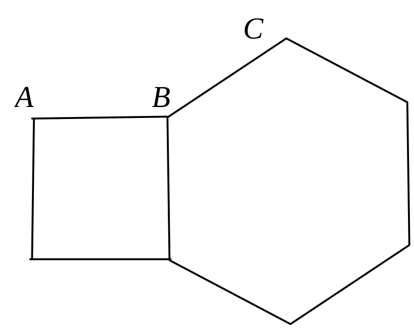
Correct answer: C

36. Point B is a mutual vertex of a regular hexagon, a square and a third regular polygon. If two of the sides of this third polygon are AB and BC, what is this polygon?

The degree measure of the interior angle of the third regular polygon is $360^\circ - 90^\circ - 120^\circ = 150^\circ$

Let n be the number of sides of the third polygon.

$\dfrac{n-2}{n}\cdot 180^\circ = 150^\circ$, $n = 12$



Correct answer: C

37. **(Tie Break No. 5)** Which of the following trig expressions is identical to $\frac{\cos(A-B)}{\sin^2 B-\cos^2 A}$?

$$\frac{\cos(A-B)}{\sin^2 B-\cos^2 A}=\frac{\cos(A-B)}{(\cos^2 A+\sin^2 A)\sin^2 B-\cos^2 A(\cos^2 B+\sin^2 B)}$$

$$=\frac{\cos(A-B)}{\sin^2 A\sin^2 B-\cos^2 A\cos^2 B}=-\frac{\cos(A-B)}{(\cos A\cos B+\sin A\sin B)(\cos A\cos B-\sin A\sin B)}$$

$$=-\frac{\cos(A-B)}{\cos(A-B)\cos(A+B)}=-\frac{1}{\cos(A+B)}=-\sec(A+B)$$

Correct answer: A

38. In a triangle ΔABC $\sin A+\cos A=\frac{\sqrt{2}}{2}$, $AC=4$, and $AB=3$, find the area of the triangle.

$\sin A\cdot\frac{1}{\sqrt{2}}+\cos A\cdot\frac{1}{\sqrt{2}}=\frac{1}{2}$, $\sin A\cdot\cos 45^\circ+\cos A\cdot\sin 45^\circ=\frac{1}{2}$, $\sin(A+45^\circ)=\frac{1}{2}$,
$A+45^\circ=150^\circ$, $A=105^\circ$

$$\sin 105^\circ=\sqrt{\frac{1-\cos 210^\circ}{2}}=\sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}=\frac{\sqrt{2+\sqrt{3}}}{2}$$

The area of the triangle is $\frac{1}{2}(AB)(AC)\sin 105^\circ=\frac{1}{2}(3)(4)\frac{\sqrt{2+\sqrt{3}}}{2}=3\sqrt{2+\sqrt{3}}$

Correct answer: B

39. Which of following sine curves has amplitude 3, period $\pi/2$, phase shift $\pi/4$, and vertical shift 1?

All the curves have amplitude 3. The sine curves of C and D have period π and sine curves of A, B, and E have period $\pi/2$. The sine curve of B have phase shift $-\pi/4$ and sine curves of A and E have phase shift $\pi/4$. The sine curve of E has vertical shift -1 and sine curve of A has vertical shift 1. Select the choice A.

Correct answer: A

40. Find the exact value of $\tan 112.5^\circ$.

$$\tan 112.5^\circ = \tan\frac{225^\circ}{2} = \frac{1-\cos 225^\circ}{\sin 225^\circ} = \frac{1-\left(-\frac{1}{\sqrt{2}}\right)}{-\frac{1}{\sqrt{2}}} = -\sqrt{2}-1$$

Correct answer: E

41. Solve the inequality $0 < \sin x + \cos x < 1$ for $0 \le x < 2\pi$.

$$0 < \sin x \cdot \frac{1}{\sqrt{2}} + \cos x \cdot \frac{1}{\sqrt{2}} < \frac{1}{\sqrt{2}},\ 0 < \sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4} < \frac{1}{\sqrt{2}},\ 0 < \sin\left(x+\frac{\pi}{4}\right) < \frac{1}{\sqrt{2}},$$

$$0 + 2k\pi < x + \frac{\pi}{4} < \frac{\pi}{4} + 2k\pi \text{ or } \frac{3\pi}{4} + 2k\pi < x + \frac{\pi}{4} < \pi + 2k\pi,$$

$-\frac{\pi}{4} + 2k\pi < x < 0 + 2k\pi$ or $\frac{\pi}{2} + 2k\pi < x < \frac{3\pi}{4} + 2k\pi$, where k is any integer.

Since $0 \le x < 2\pi$, in the first expression let $k = 1$, $\frac{7\pi}{4} < x < 2\pi$ and in the second let $k = 0$, $\frac{\pi}{2} < x < \frac{3\pi}{4}$.

Correct answer: C

42. Given that $f(2) = 5$ and $f'(x) = 2xf(x)$, find the value of $g'(\pi/3)$ if $g(x) = f(\sec x)$.

$$g'(x) = f'(\sec x)[\sec x]' = f'(\sec x)\tan x \sec x = 2\sec x f(\sec x)\tan x \sec x$$
$$= 2\sec^2 x f(\sec x)\tan x$$
$$g'(\pi/3) = 2\sec^2(\pi/3) f(\sec(\pi/3))\tan(\pi/3) = 2(2)^2 f(2)\sqrt{3} = 8\cdot 5\sqrt{3} = 40\sqrt{3}$$

Correct answer: D

43. Find $\cos\theta$ if $\cos 2\theta = -\frac{1}{9}$ and $\frac{\pi}{2} \le 2\theta \le \pi$.

$$\frac{\pi}{4} \le \theta \le \frac{\pi}{2} \text{ and } \cos\theta > 0$$

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+\left(-\frac{1}{9}\right)}{2}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

Correct answer: D

44. Find the sum of all solutions of $\sin 2\theta - 2\sin\theta = 0$, where $0 \le \theta \le 2\pi$.

$$2\sin\theta\cos\theta - 2\sin\theta = 0;\ 2\sin\theta(\cos\theta - 1) = 0$$
$$\sin\theta = 0,\quad \theta = 0,\ \pi, 2\pi$$
$$\cos\theta = 1,\quad \theta = 0, 2\pi$$
$$0 + \pi + 2\pi + 0 + 2\pi = 5\pi$$

Correct answer: C

45. Seven people sit on one row of ten seats. How many different ways can the three empty seats be distributed if there are no two empty seats next to each other and the first and last seats must be occupied?

If seven people sit on one row, there are six available empty slots between these individuals. Arranging three empty seats into these six slots without more than one empty seat in a same slot is equivalent to distributing three empty seats under the condition of the question. There are ${}_6C_3$ ways to make such arrangements.

$${}_6C_3 = \frac{6\cdot 5\cdot 4}{3\cdot 2\cdot 1} = 20$$

Correct answer: A