1. Solve the inequality $\log_{\frac{1}{5}} \frac{4x+6}{x} \geq 0$.

$\log_{\frac{1}{5}} \frac{4x+6}{x} \geq 0$ implies $0 < \frac{4x+6}{x} \leq 1$.

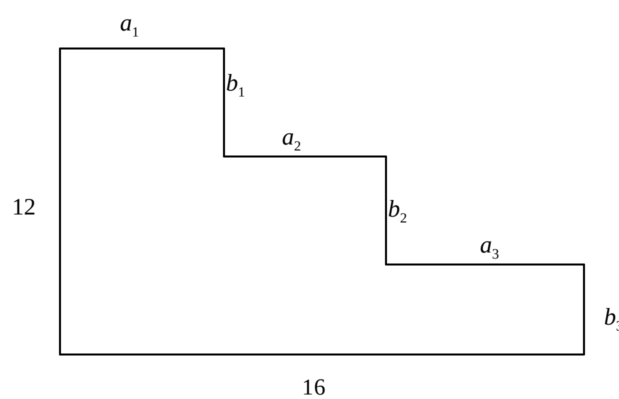
This means $0 < \frac{4x+6}{x}$ and $\frac{4x+6}{x} - 1 = \frac{3x+6}{x} \leq 0$.

The solution of $0 < \frac{4x+6}{x}$ is $\left(-\infty, -\frac{3}{2}\right) \cup (0, \infty)$ and the solution of $\frac{3x+6}{x} \leq 0$ is $[-2, 0)$.

The solution of $0 < \frac{4x+6}{x} \leq 1$ is $\left(\left(-\infty, -\frac{3}{2}\right) \cup (0, \infty)\right) \cap [-2, 0) = \left[-2, -\frac{3}{2}\right)$.

Answer: D

2. What is the perimeter of the figure shown, given that there is a right angle at each corner and that two of the sides have lengths 12 and 16 as indicated?

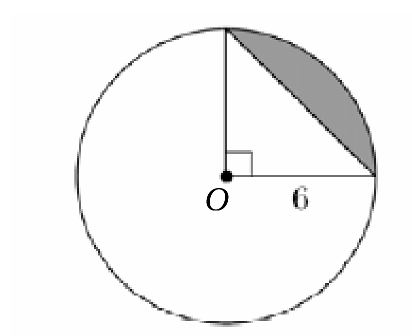


$a_1 + a_2 + a_3 = 16$ and $b_1 + b_2 + b_3 = 12$.

The perimeter is $16 + 12 + 16 + 12 = 56$.

Answer: D

3. Find the area of the shaded region in the figure if the center of the circle is O and the radius of the circle is 6.



The area of one quarter of the circle is $\frac{1}{4}(6^2)p = 9p$.

The area of the right triangle is $\frac{1}{2}(6^2) = 18$.

The area of the shaded region in the figure is $9p - 18$.

Answer: A

4. Solve the inequality: $|x^2 - 5x| < 6$

$|x^2 - 5x| < 6$ is equivalent to $-6 < x^2 - 5x < 6$.

$-6 < x^2 - 5x < 6$ implies $0 < x^2 - 5x + 6$ and $x^2 - 5x - 6 < 0$.

The solution of $0 < x^2 - 5x + 6$ is $(-\infty, 2) \cup (3, \infty)$ and the solution of $x^2 - 5x - 6 < 0$ is $(-1, 6)$.

The solution of $|x^2 - 5x| < 6$ is $((-\infty, 2) \cup (3, \infty)) \cap (-1, 6) = (-1, 2) \cup (3, 6)$.

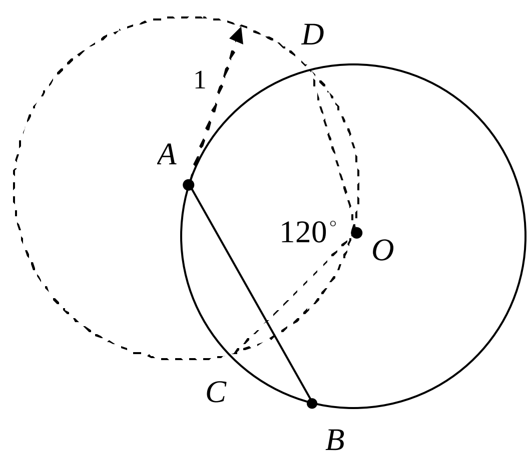
Answer: A

5. A convict escapes from a prison and has a half-hour's start on two guards and a bloodhound that race after him on his track. The guards' speed is four miles per hour. The dog's speed is 12 miles per hour. The prisoner can do only 3 miles per hour. The dog runs up to the prisoner and then back to the guards, and so on back and forth until the guards catch the prisoner. How far does the dog travel altogether?

The prisoner is 1.5 miles ahead of the guards and is running 1 mile per hour slower than the guards. It takes 1.5 hours for the guards to catch the prisoner. The bloodhound keeps running back and forth at 12 miles per hour for 1.5 hours. Therefore, the dog travels $12 \times 1.5 = 18$ miles.

Answer: D

6. Two points are picked at random on the circle $x^2 + y^2 = 1$. What is the probability that the chord joining the two points has a length of at least 1?



Let A be a point picked on the circle at random.
If the chord $AB \geq 1$, the point B must lie on the longer arc CD that does not include A. The probability of the point B lies on the longer arc CD is $\dfrac{\frac{4}{3}p}{2p} = \dfrac{2}{3}$.

Answer: D

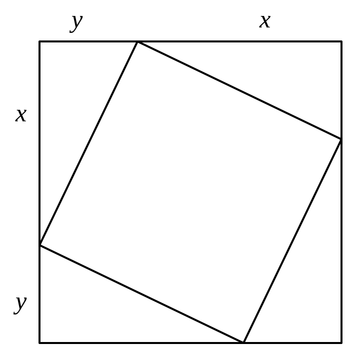
7. It is given that $\left(r+\frac{1}{r}\right)^2 = 3$. Find the value of $r^3+\frac{1}{r^3}$.

$$r^3+\frac{1}{r^3}=\left(r+\frac{1}{r}\right)\left(r^2-1+\frac{1}{r^2}\right)=\left(r+\frac{1}{r}\right)\left(\left(r^2+2+\frac{1}{r^2}\right)-3\right)=\left(r+\frac{1}{r}\right)\left(\left(r+\frac{1}{r}\right)^2-3\right)$$

$$=\left(r+\frac{1}{r}\right)\cdot(3-3)=0$$

Answer: C

8. A square is inscribed in another square, such that each vertex divides a side of the outside square into intervals of length x and y, where $x > y$. What is x/y if the area of the inscribed square is 2/3 of the area of the outside square?



The side of the large square is $x+y$ and the side of the small square is $\sqrt{x^2+y^2}$ by the Pythagorean Theorem.

$$\left(\sqrt{x^2+y^2}\right)^2 = \frac{2}{3}(x+y)^2$$

$$3(x^2+y^2) = 2(x+y)^2$$

$$x^2-4xy+y^2=0,$$

$$\left(\frac{x}{y}\right)^2-4\left(\frac{x}{y}\right)+1=0$$

$$\frac{x}{y}=2\pm\sqrt{3}$$

We choose $\frac{x}{y}=2+\sqrt{3}$ since $x>y$ and $\frac{x}{y}>1$

Answer: C

9. If $f(x)=\frac{x(x-1)}{2}$, then $f(x+2)$ equals:

$$f(x+2)=\frac{(x+2)(x+1)}{2}=\frac{(x+2)}{x}\cdot\frac{(x+1)x}{2}=\frac{(x+2)f(x+1)}{x}$$

Answer: E

10. Given three rectangles of area A, with the following dimensions, find $x-y$.

	Rectangle (i)	Rectangle (ii)	Rectangle (iii)
Length:	x	$x-3$	$x+3$
Width:	y	$y+2$	$y-1$

Solve the system

$$\begin{cases} xy = A \\ (x-3)(y+2) = A \\ (x+3)(y-1) = A \end{cases} \Rightarrow \begin{cases} xy = A \\ xy + 2x - 3y - 6 = A \\ xy - x + 3y - 3 = A \end{cases}$$

It is equivalent to $\begin{cases} 2x - 3y = 6 \\ -x + 3y = 3 \end{cases}$. Solving the system, we have $x = 9$ and $y = 4$.

Therefore, $x - y = 5$

Answer: B

11.(**Tie Break No.1**) Define the operations $a*b$ and $n\#$ as follows.

$$a*b = a + \frac{b^2 - ab}{a-b} \quad \text{and} \quad n\# = n*([n-1]*([n-2]*(\cdots*(3*(2*1)))))$$

Given that $\begin{bmatrix} 6\# & 7\# \\ 13\# & -15\# \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, find $x+y$.

$$a*b = a + \frac{b^2 - ab}{a-b} = a + \frac{b(b-a)}{a-b} = a - b$$

$n\# = n*([n-1]*([n-2]*(\cdots*(3*(2*1))))) = n*([n-1]\#) = n - [n-1]\#$

$2\# = 2*1 = 2-1 = 1$, $3\# = 3*2\# = 3-1 = 2$, $4\# = 4*3\# = 4-2 = 2$, $5\# = 5*4\# = 5-2 = 3$,

$6\# = 6*5\# = 6-3 = 3$, …

By mathematical induction, we can easily prove $[2k]\# = k$ and $[2k+1]\# = k+1$, ($k = 1, 2, 3, \ldots$).

Therefore, $\begin{bmatrix} 6\# & 7\# \\ 13\# & -15\# \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$ is equivalent to $\begin{bmatrix} 3 & 4 \\ 7 & -8 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$.

Solving the system, we have $x = 1$ and $y = \frac{3}{4}$. Thus, $x + y = \frac{7}{4}$.

Answer: D

12. If x is a real number and $4^x + 4^{-x} = 7$, then find the value of $8^x + 8^{-x}$.

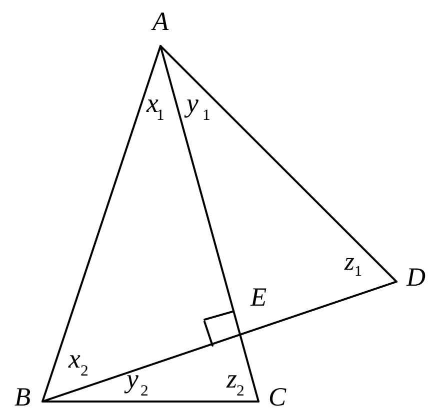
$(2^x + 2^{-x})^2 = 4^x + 2 \cdot 2^x \cdot 2^{-x} + 4^{-x} = 4^x + 2 + 4^{-x} = 7 + 2 = 9$

$2^x + 2^{-x} = 3$ since x is a real number.

$8^x + 8^{-x} = (2^x)^3 + (2^{-x})^3 = (2^x + 2^{-x})(4^x - 2^x \cdot 2^{-x} + 4^{-x}) = 3(7-1) = 18$

Answer: A

13. (**Tie Break No.2**) Triangles ΔABC and ΔABD are isosceles with $AC = AB = BD$, and the side BD intersects the side AC at E. If BD is perpendicular to AC, then $\angle C + \angle D$ is



Let $\angle BAE = x_1$, $\angle EAD = y_1$, $\angle D = z_1$, $\angle ABE = x_2$, $\angle EBC = y_2$, and $\angle C = z_2$

$AB = BD$ implies $z_1 = x_1 + y_1$ (1)

$AB = AC$ implies $z_2 = x_2 + y_2$ (2)

$BD \perp AC$ implies $z_1 + y_1 = 90^\circ$ (3)

$z_2 + y_2 = 90^\circ$ (4)

$x_1 + x_2 = 90^\circ$ (5)

Adding the equations (1), (2), (3), and (4), we have

$2(z_1 + z_2) + (y_1 + y_2) = 180^\circ + (x_1 + x_2) + (y_1 + y_2)$

$2(z_1 + z_2) = 180^\circ + (x_1 + x_2)$ (6)

Substituting (5) into (6),

$2(z_1 + z_2) = 180^\circ + 90^\circ = 270^\circ$

$\angle C + \angle D = z_1 + z_2 = 135^\circ$

Answer: D

14. If $x^2 - x = 5$, then $x^4 - x = ?$

$x^4 - x = (x^4 - 2x^3 + x^2) + (2x^3 - 2x^2) + (x^2 - x) = (x^2 - x)^2 + 2x(x^2 - x) + (x^2 - x) =$

$= 5^2 + 2x \cdot 5 + 5 = 30 + 10x = 10(3 + x)$

Answer: D

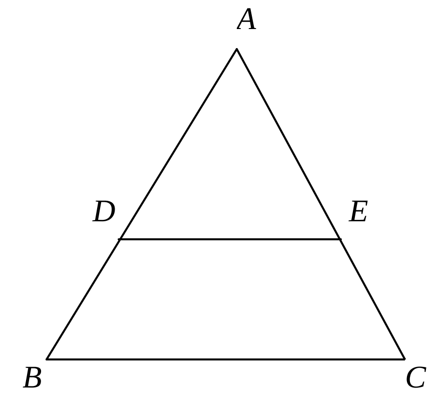
15. Which of the following polynomials $p(x)$ has the property that $\sqrt{5} - \sqrt{3}$ is a solution to the equation $p(x) = 0$?

Noticing that $\left(\sqrt{5} - \sqrt{3}\right)^2 = 8 - 2\sqrt{15}$ and $x^4 - 16x^2 + 4 = \left(x^2 - 8\right)^2 - 60$, we have

$$\left(\sqrt{5} - \sqrt{3}\right)^4 - 16\left(\sqrt{5} - \sqrt{3}\right)^2 + 4 = \left(\left(\sqrt{5} - \sqrt{3}\right)^2 - 8\right)^2 - 60 = \left(-2\sqrt{15}\right)^2 - 60 = 0$$

Answer: B

16. In the figure on the right side, $AD = 3$, $DB = 5$, $BC = 7$ and DE is parallel to BC. Find DE.



$\Delta ADE \sim \Delta ABC$

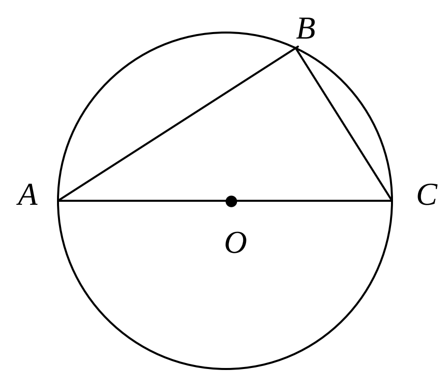
$$\frac{DE}{AD} = \frac{BC}{AB}$$

$$\frac{DE}{3} = \frac{7}{5+3}$$

$$DE = \frac{21}{8}$$

Answer: D

17. Let AC be a diameter of the circle O with a radius 5. B is a point on the circle and $BC = 4$. Find AB.



ΔABC is a right triangle with $\angle B = 90^\circ$.
$AC = 10$ and $BC = 4$
$AB = \sqrt{10^2 - 4^2} = 2\sqrt{21}$

Answer: B

18. Let $f(x) = \dfrac{15x - 1}{3x^4 - 4x^3 - 2x^2 + x}$. What is the number of vertical asymptotes with equation $x = a\ (a > 0)$ of $f(x)$?

Let $q(x) = 3x^4 - 4x^3 - 2x^2 + x = x(3x^3 - 4x^2 - 2x + 1)$ and $q_1(x) = 3x^3 - 4x^2 - 2x + 1$.
$\lim_{x \to -\infty} q_1(x) = -\infty < 0$, $q_1(0) = 1 > 0$, $q_1(1) = -2 < 0$, and $\lim_{x \to \infty} q_1(x) = \infty > 0$
Therefore, $q_1(x)$ has exact two real zeros in the interval $(0, \infty)$ and so does $q(x)$.
The maximum possible number of vertical asymptotes with equation $x = a\ (a > 0)$ of $f(x)$ is 2.
Since $q(1/15) = \dfrac{956}{16875} \neq 0$, the number of vertical asymptotes with equation $x = a\ (a > 0)$ of $f(x)$ is exactly 2.

Answer: C

19. If $x+y+z=10$, $x^2+y^2+z^2=16$, and $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=21$, find xyz.

$$(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+zx)$$
$$xy+yz+zx=\frac{(x+y+z)^2-(x^2+y^2+z^2)}{2}=\frac{10^2-16}{2}=42$$
$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{yz+zx+xy}{xyz}$$

$$xyz=\frac{yz+zx+xy}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}=\frac{42}{21}=2$$

Answer: A

20. What is the surface area of a regular tetrahedron having edge length of 4?

The surface area of the regular tetrahedron is the sum of the areas of four equilateral triangles with side 4.

$$A=4\left(\frac{1}{2}\cdot 4\cdot 2\sqrt{3}\right)=16\sqrt{3}$$

Answer: C

21. (**Tie Break No. 3**) Let $f(x)$ be a real-valued function with inverse given by $f^{-1}(x)=x(4+x^2)+2(6+x^2)$. What is the value of $f\left(f^{-1}\left(f(4)\right)\right)$?

$f^{-1}(f(4))=4$

$f(f^{-1}(f(4)))=f(4)$

$f^{-1}(x)=x(4+x^2)+2(6+x^2)=x^3+2x^2+4x+12$

Let $y=f(4)$

Solve $y^3+2y^2+4y+12=4$

$y^3+2y^2+4y+8=0$

$y^2(y+2)+4(y+2)=0$

$(y^2+4)(y+2)=0$

y is a real number. $y=-2$

$f(f^{-1}(f(4)))=f(4)=y=-2$

Answer: D

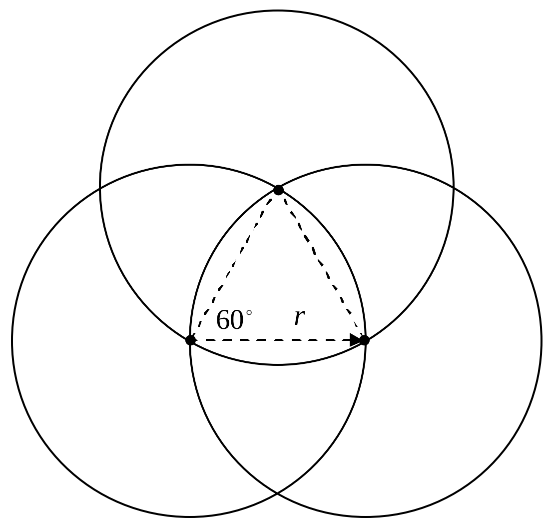
22. Let $f(x) = (x-a)(x-b)$, where a and b are real numbers. If $f(x)$ has a minimum on the interval $(-1, 1)$, which of the following must be true about a and b?

$$f(x) = (x-a)(x-b) = x^2 - (a+b)x + ab = \left(x - \frac{a+b}{2}\right)^2 + ab - \left(\frac{a+b}{2}\right)^2$$

The graph of $f(x)$ is a parabola that opens upward and $f(x)$ has a minimum on $(-1, 1)$ only if its vertex is on $(-1, 1)$. Therefore, $-1 < \frac{a+b}{2} < 1$.

Answer: D

23. Three circles have the same radius r. Each circle's center is on the two other circles. Find the area of the region common to all three circles.



The area of one sixth of the circle is $\frac{1}{6}p r^2$.

The area of the equilateral triangle with side r is

$$\frac{1}{2} \cdot r \cdot \frac{\sqrt{3}}{2} r = \frac{r^2\sqrt{3}}{4}$$

The area of the region covered by all these three circles is

$$3\left(\frac{1}{6}p r^2 - \frac{r^2\sqrt{3}}{4}\right) + \frac{r^2\sqrt{3}}{4} = \frac{p - \sqrt{3}}{2} r^2$$

Answer: A

24. Solve the equation $3^{2x} + 3^{x+1} - 4 = 0$ in the real number system.

$(3^x)^2 + 3(3^x) - 4 = 0$

$(3^x - 1)(3^x + 4) = 0$

Since x is a real number, $3^x = 1$.

Therefore, $x = 0$.

Answer: D

25. The graph of which of the following trig functions has amplitude 2 and period $4p$?

	Amplitude	Period
A. $y = 2\sin(2x)$	2	$\frac{2p}{2} = p$
B. $y = \sin(2x - 4)$	1	$\frac{2p}{2} = p$
C. $y = 2\tan\left(\frac{1}{2}x\right)$	None	$\frac{p}{1/2} = 2p$
D. $y = -2\cos\left(\frac{1}{2}x\right)$	2	$\frac{2p}{1/2} = 4p$
E. $y = 2\sin(2x + 4)$	2	$\frac{2p}{2} = p$

Answer: D

26. The graph of function $y = f(x)$ is symmetric to the graph of $y = e^{3x}$ with respect to the straight line $y = x$. Which of the following statements is true?

$f(x)$ is the inverse function of $y = e^{3x}$.

Solve $x = e^{3y}$ for y.

$\ln x = \ln e^{3y}$

$\ln x = 3y$

$y = \frac{1}{3}\ln x = f(x)$

$f(3x) = \frac{1}{3}\ln(3x) = \ln\sqrt[3]{3} + \ln\sqrt[3]{x}$

Answer: E

27. Given $f(x+1) = x^2 - 3x + 2 \quad (-\infty < x < 3/2)$, find the domain of the function $y = f^{-1}(x)$.

Let $u = x + 1$.

$x = u - 1$

$f(u) = (u-1)^2 - 3(u-1) + 2 \quad (-\infty < u - 1 < 3/2)$

$f(u) = u^2 - 5u + 6 \quad (-\infty < u < 5/2)$

$f(u) = \left(u - \frac{5}{2}\right)^2 - \frac{1}{4} \quad (-\infty < u < 5/2)$

The range of the function $f(x)$ is $(-1/4, \infty)$

The domain of its inverse $f^{-1}(x)$ is $(-1/4, \infty)$.

Answer: D

28. For what real values of x is $\left|x^2+3x+2\right| = x^2+3x+2$?

The equality is true when $x^2+3x+2 \geq 0$.
Solve $x^2+3x+2 \geq 0$.
$(x+1)(x+2) \geq 0$
$x \leq -2$ or $-1 \leq x$

Answer: B

29. Find the domain of the composite function $f \circ g$ where $f(x) = \dfrac{5}{x+5}$ and $g(x) = -\dfrac{5}{x}$.

$$(f \circ g)(x) = \frac{5}{\left(-\frac{5}{x}\right)+5}$$

We have to have $x \neq 0$ and $\left(-\frac{5}{x}\right)+5 \neq 0$, that is $x \neq 1$.
The domain of $f \circ g$ is $\{x \mid x \neq 0, x \neq 1\}$.

Answer: D

30. Which of the following is NOT a true statement for all real j ?

If $j = \frac{p}{6}$, $1+\cot^2\frac{p}{6} = 1+\left(\sqrt{3}\right)^2 = 4$ and $\sec^2\frac{p}{6} = \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}$.
$1+\cot^2 j = \sec^2 j$ is not true for $j = \frac{p}{6}$.

Answer: C

31. Find the following limit where n is an integer: $\lim_{n\to\infty}\frac{1+2+3+\cdots+2n}{n^2+1}$

$$\lim_{n\to\infty}\frac{1+2+3+\cdots+2n}{n^2+1}=\lim_{n\to\infty}\frac{\frac{2n(2n+1)}{2}}{n^2+1}=\lim_{n\to\infty}\frac{2n^2+n}{n^2+1}=\lim_{n\to\infty}\frac{2n^2}{n^2}=2$$

Answer: A

32. (**Tie Break No.4**) Find a vertical line $x=k$ that divides the area enclosed by $x=\sqrt{y}$, $x=2$, and $y=0$ into two equal parts.

Solve $\int_0^k x^2dx=\int_k^2 x^2dx$ for k.

$$\left.\frac{x^3}{3}\right|_0^k=\left.\frac{x^3}{3}\right|_k^2$$

$$\frac{k^3}{3}=\frac{2^3}{3}-\frac{k^3}{3}$$

$2k^3=8$ and $k=\sqrt[3]{4}$.

Answer: A

33. Find $\frac{d}{dx}[f(x)]$ if $\frac{d}{dx}[f(3x)]=6x$.

We have $\frac{d}{dx}[f(3x)]=f'(3x)\cdot\frac{d}{dx}[3x]=3f'(3x)$.

$$3f'(3x)=6x$$

$$f'(3x)=\frac{2}{3}(3x)$$

$$f'(x)=\frac{2}{3}x$$

Answer: E

34. Express $\dfrac{\sin x}{1-\cos x}-\dfrac{\sin x}{1+\cos x}$ in terms of a single trigonometric function.

$$\frac{\sin x}{1-\cos x}-\frac{\sin x}{1+\cos x}=\frac{\sin x(1+\cos x)-\sin x(1-\cos x)}{1-\cos^2 x}$$

$$=\frac{2\sin x\cos x}{\sin^2 x}=\frac{2\cos x}{\sin x}=2\cot x$$

Answer: D

35. Find the distance from a highest point to its closest lowest point on the graph of $y=\cos x$.

The coordinates of the highest points on the graph of $y=\cos x$ are $(2kp,1)$, where k is any integer.
The lowest point that is closest to $(2kp,1)$ is $((2k-1)p,-1)$ or $((2k+1)p,-1)$.
The distance between them is $\sqrt{(2kp-(2k\pm1)p)^2+(1-(-1))^2}=\sqrt{p^2+4}$.

Answer: E

36. The two shorter sides of a right triangle have lengths 2 and $\sqrt{5}$. Let x be the smallest angle of the triangle. What is $\cos x$?

The hypotenuse of the right triangle is $\sqrt{2^2+\left(\sqrt{5}\right)^2}=3$. The adjacent side of the smallest angle x is $\sqrt{5}$. Therefore, $\cos x=\dfrac{\sqrt{5}}{3}$.

Answer: D

37. If we have $f(\sin x) = 5 - \cos 4x$, then $f(\cos x) = ?$

$$f(\sin x) = 5 - \cos 4x = 5 - \left(2\cos^2 2x - 1\right) = 5 - \left(2\left(1 - 2\sin^2 x\right)^2 - 1\right)$$
$$f(\cos x) = 5 - \left(2\left(1 - 2\cos^2 x\right)^2 - 1\right) = 5 - \left(2\left(2\cos^2 x - 1\right)^2 - 1\right) = 5 - \left(2\cos^2 2x - 1\right) = 5 - \cos 4x$$

Answer: A

38. (**Tie Break No. 5**) Solve the inequality $0 < \sin x + \cos x < 1$ for $0 \le x < 2p$.

$$0 < \sqrt{2}\left(\sin x \cdot \frac{1}{\sqrt{2}} + \cos x \cdot \frac{1}{\sqrt{2}}\right) < 1$$
$$0 < \sin x \cos\frac{p}{4} + \cos x \sin\frac{p}{4} < \frac{1}{\sqrt{2}}$$
$$0 < \sin\left(x + \frac{p}{4}\right) < \frac{1}{\sqrt{2}}$$
$$\ldots,\ 0 < x + \frac{p}{4} < \frac{p}{4},\ \frac{3p}{4} < x + \frac{p}{4} < p,\ 2p < x + \frac{p}{4} < \frac{9p}{4},\ \frac{11p}{4} < x + \frac{p}{4} < 3p,\ldots$$
$$\ldots,\ -\frac{p}{4} < x < 0,\ \frac{p}{2} < x < \frac{3p}{4},\ \frac{7p}{4} < x < 2p,\ \frac{5p}{2} < x < \frac{11p}{4},\ldots$$

Since $0 \le x < 2p$, we have $\frac{p}{2} < x < \frac{3p}{4}$ or $\frac{7p}{4} < x < 2p$.

Answer: C

39. $\sin\left(\frac{p}{12}\right) = ?$

$$\sin\left(\frac{p}{12}\right) = \sin\left(\frac{p}{4} - \frac{p}{6}\right) = \sin\frac{p}{4}\cos\frac{p}{6} - \cos\frac{p}{4}\sin\frac{p}{6} = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

Answer: A

40. Using degree measure, evaluate the following sum:
$\sin^{-1}(\sin 100^\circ) + \cos^{-1}(\cos 100^\circ) + \tan^{-1}(\tan 100^\circ)$

$$\begin{aligned}&\sin^{-1}(\sin 100^\circ) + \cos^{-1}(\cos 100^\circ) + \tan^{-1}(\tan 100^\circ)\\ &= \sin^{-1}(\sin 80^\circ) + \cos^{-1}(\cos 100^\circ) + \tan^{-1}(\tan(-80^\circ))\\ &= 80^\circ + 100^\circ + (-80^\circ) = 100^\circ\end{aligned}$$

Answer: A

41. A bowling ball with a circumference of 30 inches is tightly packed into the smallest possible cubical box for shipment. What proportion of the box's space is occupied by the bowling ball?

Let r be the radius of the ball.

The volume of the ball is $V_1 = \frac{4}{3} p r^3$.

The edge of the cubical box is $2r$ and the volume of the box is $V_2 = (2r)^3 = 8r^3$.

$$\frac{V_1}{V_2} = \frac{\frac{4}{3} p r^3}{8r^3} = \frac{p}{6}$$

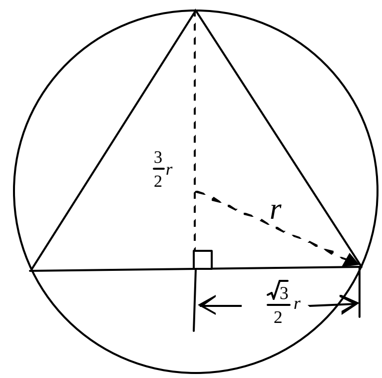
Answer: B

42. There are 1817 freshmen at Mullet University this semester. Of these, 458 are taking at least one course in computer science, 571 are taking at least one course in mathematics, and 300 are taking courses in both computer science and mathematics. How many of these freshmen are not taking a course in either computer science or mathematics this semester?

The number of the students not taking either is
$1817 - (458 + 571 - 300) = 1088$

Answer: D

43. A right circular cone is inscribed in a sphere with a radius *r*. If the area of the base of the cone is $\frac{3p r^2}{4}$. Find the volume of the cone.



The radius of the base of the cone is $R = \sqrt{\dfrac{\frac{3p r^2}{4}}{p}} = \dfrac{\sqrt{3}}{2}r$.

The height of the cone is $h = r + \sqrt{r^2 - \left(\dfrac{\sqrt{3}}{2}r\right)^2} = r + \dfrac{1}{2}r = \dfrac{3}{2}r$.

The volume of the cone is

$$V = \frac{1}{3}h\left(pR^2\right) = \frac{1}{3}\cdot\left(\frac{3}{2}r\right)\cdot p\cdot\left(\frac{\sqrt{3}}{2}r\right)^2 = \frac{3p r^3}{8}$$

Answer: A

44. Find the oblique asymptote of $H(t) = \dfrac{2t^3 + 11t^2 + 5t - 1}{t^2 + 6t + 5}$

Using long division, we have

$$H(t) = 2t - 1 + \frac{t+4}{t^2 + 6t + 5}$$

The oblique asymptote of $H(t)$ is $y = 2t - 1$.

Answer: B

45. If $\cos j = 5/13$ and $270^\circ < j < 360^\circ$, find $\tan j$

$\tan j < 0$ since $270^\circ < j < 360^\circ$.

$$\tan j = -\sqrt{\sec^2 j - 1} = -\sqrt{\frac{1}{\cos^2 j} - 1} = -\sqrt{\left(\frac{13}{5}\right)^2 - 1} = -\sqrt{\frac{169}{25} - 1} = -\sqrt{\frac{144}{25}} = -\frac{12}{5}$$

Answer: B