

**Math Bowl 2006 Written Test**

1. If  $xy = -3$  and  $x^2 + y^2 = 12$ , then  $(x - y)^2 = ?$
- A. 25  
B. 18  
C. 16  
D. 4  
E. 11
2. In  $\triangle ABC$ ,  $m\angle C = 120^\circ$ ,  $a = 3$ , and  $b = 5$ . Find  $m\angle A$
- A.  $\sin^{-1}\left(\frac{3\sqrt{57}}{38}\right)$   
B.  $\sin^{-1}\left(\frac{5\sqrt{57}}{38}\right)$   
C.  $\sin^{-1}\left(\frac{3\sqrt{3}}{14}\right)$   
D.  $\sin^{-1}\left(\frac{5\sqrt{3}}{14}\right)$   
E. None of the above
3. A regular hexagon and an equilateral triangle have the same perimeter. What is the ratio of the area of the hexagon to the area of the triangle?
- A. 2:3  
B. 4:1  
C. 3:2  
D. 1:4  
E. 5:3

4. The period of a simple pendulum is directly proportional to the square root of its length. If a pendulum has a length of 6 feet and a period of 2 seconds, to what length should it be shortened to achieve a 1 second period?

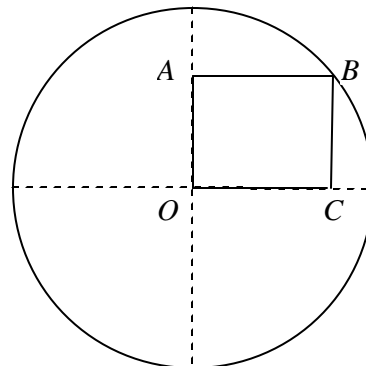
- A. 2.5 ft.
- B. 1.5 ft
- C. 2.0 ft.
- D. 3.5 ft.
- E. 0.5 ft.

5. If  $x^2 - x - 1$  divides  $ax^6 + bx^5 + 1$  evenly, find the sum of  $a$  and  $b$ .

- A. 3
- B. -2
- C. 5
- D. -5
- E. 0

6. Given a circle of radius  $r$  with a rectangle  $ABCO$  inscribed in one quadrant, what is the length of diagonal  $AC$ ?

- A.  $\frac{3p}{4}r^2$
- B.  $\frac{2p}{3}r$
- C.  $\frac{p}{4}r^2$
- D.  $r + \frac{p}{8}$
- E.  $r$



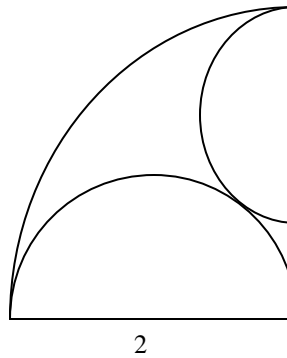
7. The number of horses and sheep in a certain town is such that the ratio of the difference to the sum is 1:7 and the ratio of the sum to the product is 7:24. What is the sum of the number of horses and sheep?
- A. 20
  - B. 14
  - C. 10
  - D. 17
  - E. 24
8. Tom can plow a field in 12 hours, but with Pat helping him they can plow the field together in 8 hours. If Pat works alone plowing for 12 hours, how long will it take Tom working alone to plow the remainder of the field?
- A. 3 hours
  - B. 4 hours
  - C.  $4\frac{1}{2}$  hours
  - D. 6 hours
  - E.  $6\frac{2}{3}$  hours
9. The will of an eccentric millionaire reads as follows: “I leave  $\frac{4}{17}$  of my estate to my son,  $\frac{7}{13}$  of the remainder to my wife,  $\frac{2}{3}$  of this remainder to my daughter, and the remaining \$2,000,000 to my dog.” What was the total amount of the estate?
- A. \$10,000,000
  - B. \$20,000,000
  - C. \$15,000,000
  - D. \$18,000,000
  - E. \$17,000,000

10. A survey of 40 shoppers found that 12 liked Brand X, 23 liked Brand Y, and twice as many disliked both brands as liked both brands. How many liked both brands?

- A. 3
- B. 8
- C. 2
- D. 5
- E. 10

11. **(Tie Break No.1)** The radius of the quarter-circle is 2. The two semi-circles are tangent to each other. Find the radius of the smaller semicircle.

- A.  $1/2$
- B.  $2/3$
- C.  $3/4$
- D.  $1/4$
- E. 1



12. In a right triangle  $\triangle ABC$ ,  $C = 90^\circ$ . Which of the following statements is NOT true?

- A.  $0 < \sin A + \sin B \leq \sqrt{2}$
- B.  $\sin^2 A + \sin^2 B = \sin^2 C$
- C.  $\cos^2 A + \cos^2 B = 1$
- D.  $\tan \frac{A+B}{2} = \sin C$
- E. At least one of the above statements is not true.

13. **(Tie Break No.2)** What is the value of the expression

$$\frac{1}{\log_2 100!} + \frac{1}{\log_3 100!} + \frac{1}{\log_4 100!} + \cdots + \frac{1}{\log_{100} 100!} ?$$

- A. 1
- B. 0.1
- C. 0.01
- D. 10
- E. None of the above
14. If  $x^2 = x + 5$ , then  $x^3 = ?$
- A.  $6x + 5$
- B.  $x + 10$
- C.  $6x^2 + 5$
- D.  $x^2 + 5x + 5$
- E. None of the above
15. If the square of the geometric mean of  $x$  and  $y$  is equal to the sum of  $x$  and  $y$ , express  $y$  in terms of  $x$ .
- A.  $y = \frac{1}{x-1}$
- B.  $y = \frac{x}{x-1}$
- C.  $y = \frac{1}{x} + 1$
- D.  $y = \frac{1}{x^2 - 1}$
- E.  $y = \frac{x}{x+1}$

16. **(Tie Break No.3)** The library in Destin has between 1000 and 2000 books. Of these, exactly 25% are fiction,  $\frac{1}{13}$  are biographies, and  $\frac{1}{17}$  are atlases. How many books are either biographies or atlases?

- A. 240
- B. 250
- C. 270
- D. 280
- E. None of the above.

17. If  $n$  is an integer, which of the following must be true?

- A.  $3n+1$  is odd
- B.  $n(n+2)$  is even
- C.  $n(3n+3)$  is divisible by 6
- D.  $n(n+1)$  is divisible by 3
- E.  $n(n+3)$  is divisible by 4

18.  $x(x(x(x+2)+2)+2)+2 = ?$

- A.  $x^4 + 2x^3 + 2x^2 + 2x + 2$
- B.  $x^4 + 2x^3 + 4x^2 + 2x + 2$
- C.  $x^4 + 2x^3 + 8x^2 + 2x + 2$
- D.  $x^4 + 4x^3 + 16x^2 + 8x + 4$
- E. None of the above

19. Let  $ABCD$  be a square piece of paper.  $A$  is folded onto  $C$  then  $B$  is folded onto  $D$ . The area of the resulting figure is 9 square inches. Find the perimeter of the square  $ABCD$ .
- A. 9 inches
  - B. 16 inches
  - C. 18 inches
  - D. 24 inches
  - E. 36 inches
20. If  $f(x) = \sqrt[3]{x}$  and  $g(x) = x - 5$ , find  $(f^{-1} \circ g^{-1})(-3)$
- A. -6
  - B. -4
  - C. 2
  - D. -8
  - E. None of the above
21. If a 1-inch thick slab is sliced off one end of a cube, 294 cubic inches remain. What is the length of an edge of this cube?
- A.  $3 + \sqrt{5}$
  - B.  $5\sqrt{3}$
  - C. 7
  - D.  $11\sqrt{2}$
  - E. 12

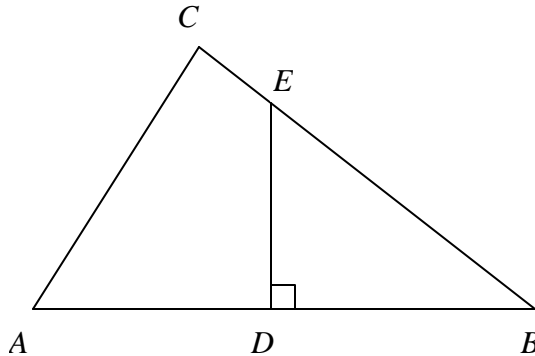
22. Let  $A = \begin{bmatrix} 1 & 1 & z \\ 0 & 1 & 0 \\ x & -1 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & -4 & -1 \\ 0 & y & 0 \\ -2 & 3 & 1 \end{bmatrix}$ , find the values of  $x$ ,  $y$ , and  $z$  so that

$B = A^{-1}$ . Then  $x + y + z = ?$

- A. 0
- B. 1
- C. -1
- D. 4
- E. None of the above

23. In the figure, it is given that angle  $C = 90^\circ$ ,  $\overline{AD} = \overline{DB}$ ,  $DE$  perpendicular to  $AB$ ,  $\overline{AB} = 20$ , and  $\overline{AC} = 12$ . Find the area of the quadrilateral  $ADEC$ .

- A. 75
- B. 58.5
- C. 48
- D. 37.5
- E. None of the above



24. Find the exact value of  $\tan\left(\cos^{-1}\frac{1}{3} - \sin^{-1}\frac{1}{3}\right)$

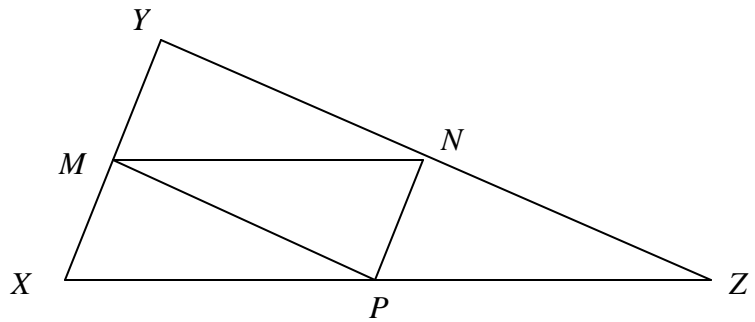
- A. 0
- B.  $\frac{7}{4\sqrt{2}}$
- C.  $4\sqrt{2}$
- D.  $\frac{4\sqrt{2}}{7}$
- E. 1

25. The sum of the solutions of  $(\log_4 x)^2 = \log_4(x^2)$  is equal to?

- A. 2
- B. 3
- C. 12
- D. 16
- E. 17

26. In  $\triangle XYZ$ , points  $M$ ,  $N$ , and  $P$  are midpoints. If  $XY = 10$ ,  $YZ = 15$ , and  $XZ = 17$ , what is the perimeter of  $\triangle MNP$ ?

- A.  $10\frac{2}{3}$
- B. 14
- C. 16
- D. 21
- E. Cannot be determined



27. If  $f(x^5) = \log_a x$ , then  $f(2) = ?$

- A.  $\log_a 2$
- B.  $\log_a 32$
- C.  $\log_a \frac{1}{32}$
- D.  $\frac{\log_a 2}{5}$
- E.  $5\log_a \frac{1}{2}$

28. Find  $3a + 2b + c$  if the parabola  $y = ax^2 + bx + c$  passes through the points  $(1, -2)$ ,  $(-2, 19)$ , and  $(3, 4)$ .

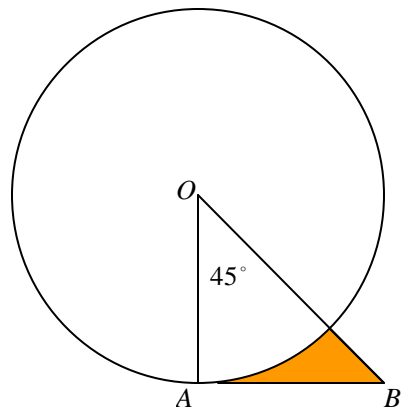
- A. 0
- B. 1
- C. -2
- D. 3
- E. -3

29. Simplify  $\left(\frac{(x+1)^2(x^2-x+1)^2}{x^3+1}\right)^2 \cdot \left(\frac{(x-1)^2(x^2+x+1)^2}{x^3-1}\right)^2$

- A. 1
- B.  $(x+1)^4$
- C.  $(x^3+1)^4$
- D.  $(x^6-1)^2$
- E.  $(x^6+1)^4$

30. In the figure, if the radius  $OA$  of the circle is 6 and  $AB$  is tangent to the circle what is the area of the shaded part?

- A.  $36 - \frac{3p}{2}$
- B.  $18 - \frac{9p}{2}$
- C.  $18 - \frac{3p}{2}$
- D.  $18 - 3p$
- E.  $36 - 9p$



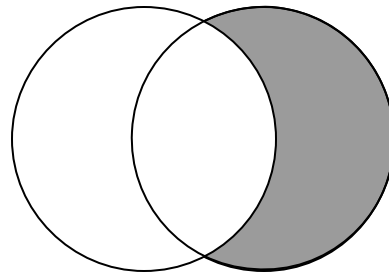
31. It is given that  $\left(r + \frac{1}{r}\right)^2 = 3$ . Find the value of  $r^3 + \frac{1}{r^3}$
- A. 1  
B. 2  
C. 0  
D. 3  
E. 6
32. A rectangular box with a square base has the property that the ratio of its volume to its surface area is 1. If one side of the square base has length  $a$  units, express the height  $b$  in terms of  $a$ .
- A.  $\frac{2a}{a-4}$   
B.  $\frac{4a}{a-2}$   
C.  $\frac{2a}{a+4}$   
D.  $\frac{4a}{a-2}$   
E. None of the above.
33. Let  $F(x) = \sqrt{\sin^4 x + 4\cos^2 x} - \sqrt{\cos^4 x + 4\sin^2 x}$ . Which of the following expressions is equivalent to  $F(x)$ ?
- A.  $1 - \sqrt{2} \sin x$   
B.  $-1 + \sqrt{2} \cos x$   
C.  $\cos x - \sin x$   
D.  $\cos 2x$   
E. None of the above

34. Find the exact value of  $\tan\left(\frac{5p}{12}\right)$ .

- A. 3
- B.  $\sqrt{3}$
- C.  $1+\sqrt{2}$
- D.  $\frac{\sqrt{3}+1}{2}$
- E.  $2+\sqrt{3}$

35. **(Tie Break No.4)** In the figure, two circles have the same radius 6 and the center of one circle is on the other one. Find the area of the shaded region.

- A.  $12p - 6\sqrt{3}$
- B.  $24p - 18\sqrt{3}$
- C.  $12p + 18\sqrt{3}$
- D.  $36p - 18\sqrt{3}$
- E.  $12p + 27\sqrt{3}$



36. How many ordered triples of positive integers  $(x, y, z)$  satisfy  $(x^y)^z = 64$

- A. 6
- B. 7
- C. 8
- D. 9
- E. None of the above

37. Given  $r = \frac{3}{4\cos q - \sin q}$ , find an equivalent equation in rectangular coordinates.

A.  $y = \frac{3}{4-x}$

B.  $4x - y = 3$

C.  $3y + 4 = -1$

D.  $3y + 4 = -x$

E. It is not possible to write the equation in rectangular coordinates.

38. **(Tie Break No.5)** Find the minimum value of the function  $f(x) = \frac{1 - \cos 2x + 4\cos^2 x}{\sin 2x}$  on the open interval  $(0, \pi/2)$

A. 3

B.  $2\sqrt{2}$

C. 2

D.  $2\sqrt{3}$

E. 4

39.  $\sin^{-1}(\sin(\mathbf{p} + 1)) =$

A. 1

B. -1

C.  $\mathbf{p} + 1$

D.  $\mathbf{p} - 1$

E. None of the above.

40. If  $x = 2\sin q$  and  $0 \leq q < \frac{\pi}{2}$ , then  $\ln |\sec q + \tan q| - \ln |\cos q| = ?$
- A.  $\ln \left| \frac{x}{\sqrt{4-x^2}} \right|$
- B.  $\ln \left| \frac{4}{\sqrt{4-x^2}} \right|$
- C.  $\ln \left| \frac{2x}{\sqrt{4-x^2}} \right|$
- D.  $\ln \left| \frac{2}{2-x} \right|$
- E. None of the above
41. Let  $F(x) = g(3x^2)$  and  $g(x)$  be a differentiable function. If  $g'(12) = 5$  and  $g(12) = -3$ , find  $F'(2)$ .
- A. 5
- B. -3
- C. 60
- D. -36
- E. None of the above
42. If  $\cos x = \frac{3}{5}$  and  $\cot x$  is negative, find the value of  $\frac{\sin x - \tan x}{1 + \sec x}$ .
- A. 0
- B.  $\frac{1}{5}$
- C.  $-\frac{4}{5}$
- D.  $\frac{4}{5}$
- E.  $-\frac{1}{5}$

43. The oracle of Niceville tells the truth whenever she chooses to answer a question, and the probability that she will choose to answer a question is  $\frac{1}{3}$ . A student plans to ask the oracle 3 times for his grade on the semester exam. What is the probability that he will get the answer?
- A.  $\frac{2}{3}$
- B.  $\frac{19}{27}$
- C.  $\frac{5}{9}$
- D. 1
- E.  $\frac{7}{8}$
44. The perimeter of a rectangle is 100 and its diagonal has length  $x$ . What is the area of the rectangle?
- A.  $625 - x^2$
- B.  $625 - \frac{x^2}{2}$
- C.  $1250 - x^2$
- D.  $1250 - \frac{x^2}{2}$
- E. None of the above
45. The area of the circle formed by the intersection of a sphere and a plane is  $p$ . If the distance from the center of the sphere to the plane is 1, find the surface area of the sphere.
- A.  $8\sqrt{2}p$
- B.  $4\sqrt{2}p$
- C.  $8p$
- D.  $4p$
- E.  $16p$