15th ANNUAL

Okaloosa-Walton Community College

MATHEMATICS TOURNAMENT

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Niceville, Florida

Test Booklet

INSTRUCTIONS: **This is a 90 minute, 45 problem, multiple-choice examination.** There are five (5) possible responses to each question or problem. You are to **select the one (1) best answer to each.** You may mark on the test booklet, and the back of each page may be used for additional work space. Darken completely the circle below the letter of your response to each question on your score sheet. Your student number is encoded on your score sheet for you. Mark your answers **boldly** with a No. 2 pencil. If you must change an answer, completely erase your first choice and then record the new answer. Incomplete erasures and multiple marks for any question will be scored as an incorrect response. Do not mark beyond question 45. Your score will be computed by the following formula: Score = 45 + (4C - I) , where C represents the number of correct answers and I represents the number of incorrect answers. If you can definitely rule out at least one choice, it will be in your favor to randomly guess from the remaining choices. There is no penalty for problems left unanswered. In the event of a tie, the indicated tie-breaker questions will be checked in order until the tie is broken.

Review and check your score sheet carefully. Your student identification number has been encoded on your answer sheet and it has been checked by our marked-sense card reader. If you alter this number in any way you may **disqualify yourself and your team from consideration for any awards.**

When you complete your test, close your test booklet, turn your answer sheet over, and sit quietly until all of the answer sheets are collected. You may keep your pencil and your test booklet. **Calculators are Not Allowed!**

PLEASE DO NOT OPEN
UNTIL INSTRUCTED TO DO SO

1. I mistakenly divided a number by 5 when I should have multiplied by 5. Find my error as a percentage of the correct value.

   A. 50%

   B. 96%

   C. 68%

   D. 75%

   E. 30%

2. If $f(x) = \log_a x$, then $-f(x) = ?$

   A. $f(-x)$

   B. $f^{-1}(x)$

   C. $[f(x)]^{-1}$

   D. $\log_x a$

   E. $\log_{1/a} x$

3. Find the inverse function of $f(x) = e^{-x} + 1 \quad (0 < x)$

   A. $y = \ln \frac{1}{x-1} \qquad (1 < x < \infty)$

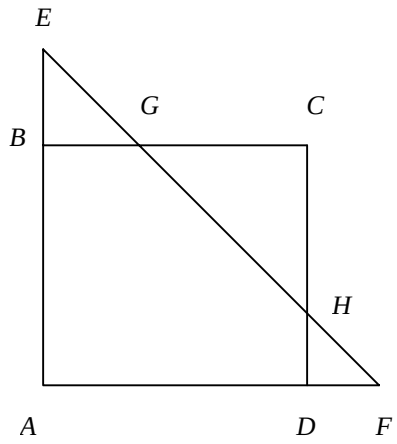
   B. $y = -\ln \frac{1}{x-1} \quad (1 < x < \infty)$

   C. $y = \ln \frac{1}{x-1} \qquad (1 < x < 2)$

   D. $y = \ln \frac{1}{x-1} \qquad (1 < x \le 2)$

   E. $y = -\ln \frac{1}{x-1} \quad (1 < x < 2)$

4. In the figure, $ABCD$ is a rectangle. $AB = 6,\ AD = 8,$ and $AE = AF = 10$. Find the area of ΔGCH.



A. 8

B. 10

C. 6

D. 12

E. 9

5. $\sec\left(\tan^{-1}\dfrac{x}{2}\right) = ?$

A. $\dfrac{\sqrt{x^2+4}}{2}$

B. $\dfrac{\sqrt{x^2+4}}{x}$

C. $\dfrac{x^2+4}{2}$

D. $\dfrac{x}{\sqrt{x^2+4}}$

E. $\dfrac{2}{\sqrt{x^2+4}}$

6. A pyramid with a square base is inscribed in a right circular cylinder whose base radius is 4 and height is 9. What is the volume of the pyramid?

A. 48

B. 144

C. 288

D. 96

E. 108

7. **(Tie Break 1)** $f(x)$ is an odd function and $g(x)$ is an even function. $f(x)+g(x)=\frac{1}{x-1}$. Find $f(-2)$

A. $2/3$

B. $3/2$

C. $-2/3$

D. $-3/2$

E. Insufficient information given

8. If the solution set of the inequality $3x^2 - ax + b < 0$ is $\left\{x \mid -\frac{2}{3} < x < \frac{5}{3}\right\}$, then the value of *a* is

A. 3

B. -3

C. $-10/3$

D. 1

E. Insufficient information given

9. Three circles of radii 3, 4, and 7 tangent to each other externally. Find the angle *A*.

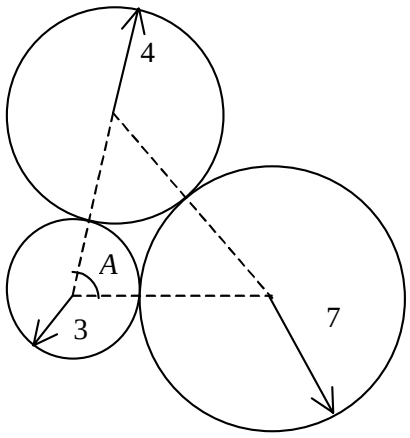
A. $\cos^{-1}(5/11)$

B. $\cos^{-1}(1/5)$

C. $\cos^{-1}(7/10)$

D. $\cos^{-1}(3/5)$

E. Insufficient information given



10. If $\log_2(\log_3(\log_4 x)) = \log_3(\log_4(\log_2 y)) = \log_4(\log_2(\log_3 z)) = 0$, then $x + y + z = ?$

A. 50

B. 58

C. 89

D. 111

E. None of these

11. Let $f(x) = \log\left(\frac{1+x}{1-x}\right)$ for $-1 < x < 1$. Then $f\left(\frac{3x + x^3}{1+3x^2}\right) = ?$

A. $-f(x)$

B. $2f(x)$

C. $[f(x)]^3$

D. $3f(x)$

E. None of these

12. An aquarium on a level table has rectangular faces and is 10 inches wide and 8 inches high. When it is tilted and the water surface reaches the right top edge, only three-fourths of the rectangular bottom is covered by the water. The depth of the water when the bottom is level would be

A. 2.5 inches

B. 3 inches

C. 3.25 inches

D. 4 inches

E. None of these

13. Square *MATH* is inscribed in circle *C* and square *BOWL* is inscribed in a semicircle of circle *C*. What is the ratio of the areas of *MATH* to *BOWL*?

A. 2:1

B. 3:1

C. 5:2

D. 5:3

E. None of these

14. Find the sum of the solutions of the equation $18e^{2x} - 27e^{x} + 10 = 0$.

A. $\ln(2/3)$

B. $\ln(3/2)$

C. $\ln(5/9)$

D. $\ln(9/5)$

E. $\ln(2.7)$

15. If $\tan\alpha = 2/3$, and the terminal side of α lies in quadrant III, then $\sin 2\alpha =$?

A. $-\frac{4}{\sqrt{13}}$

B. $-\frac{12}{13}$

C. $-\frac{3}{\sqrt{13}}$

D. $\frac{2}{\sqrt{13}}$

E. $\frac{12}{13}$

16. **(Tie Break 2)** A circle is inscribed inside the parabola $y = 2x^2$ with points of tangency at $x = \pm 1$. Find the *y*-coordinate of the point of the circle nearest the *x*-axis.

A. $\frac{4}{5}$

B. $\frac{5}{4}$

C. $\frac{17}{16}$

D. $\frac{9-\sqrt{17}}{4}$

E. $\frac{9}{4}$

17. Find *a*, *b*, *c* of the function $y = ax^2 + bx + c$ whose graph contains the points $(1, 2)$, $(-2, -7)$, and $(2, -3)$.

A. $a = 2,\ b = -1,\ c = 3$

B. $a = 2,\ b = -1,\ c = 3$

C. $a = 3,\ b = 1,\ c = -2$

D. $a = 4,\ b = 1,\ c = 3$

E. $a = -2,\ b = 1,\ c = 3$

18. What is the area of the triangle formed by the *x*-axis, the *y*-axis, and the tangent line to $y = -2x^2 + 6$ at the point $(3, -12)$?

A. 3

B. 12

C. 18

D. 24

E. 27

19. For what values of x is $\left|x^2+3x+2\right| = x^2+3x+2$ true?

A. All real numbers

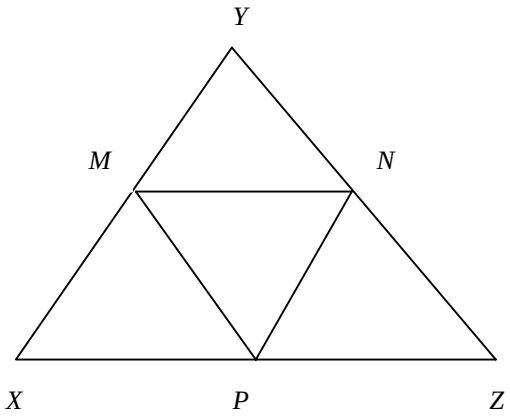
B. $x \le -2$ or $x \ge -1$

C. $-2 < x < -1$

D. $x \ge 0$

E. None of the above

20. In ΔXYZ, points M, N, and P are midpoints. If $XY = 10$, $YZ = 15$, and $XZ = 17$, what is the perimeter of ΔMNP?



A. $10\frac{2}{3}$

B. 14

C. 16

D. 21

E. Insufficient information given

21. If the roots of the equation $x^2+bx+c=0$ are r and s, the value of $\frac{(r+s)^2}{rs}$ in terms of b and c is

A. $\frac{b^2}{c^2}$

B. $\frac{c^2}{b}$

C. $\frac{b^2}{c}$

D. $-\frac{b}{c}$

E. $-\frac{b^2}{c}$

22. **(Tie Break 3)** If $45^\circ < A < B < 90^\circ$, $\sin A + \cos A = a$ and $\sin B + \cos B = b$, then we have

A. $a < b$

B. $a > b$

C. $ab < 1$

D. $ab > 2$

E. None of these

23. Alex can run around a circular track in 40 seconds. Bonnie, running in the opposite direction, meets Alex every 15 seconds. What is Bonnie's time to run around the track, expressed in seconds?

A. 24

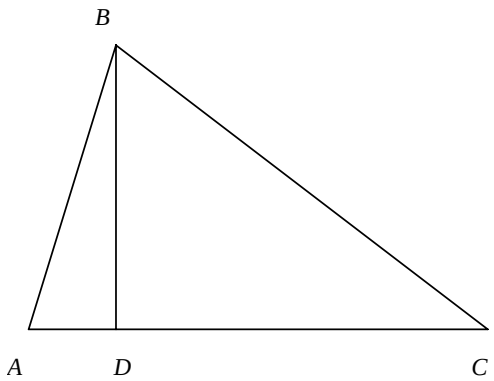
B. 26

C. 30

D. 35

E. 38

24. In the diagram below, $\overline{BD}$ is perpendicular to $\overline{AC}$. $AB = 3$, $BC = 7$, and $AC = 9$. What is BD?



A. $\frac{13}{18}$

B. $\frac{41}{18}$

C. $\frac{\sqrt{1235}}{18}$

D. $9 - \left(\frac{41}{18}\right)^2$

E. 2.5

25. Twenty men did $1/4$ of a job in 8 days. Then, because of a building dedication, it became necessary to complete the job in the next 5 days. How many additional men were added to the crew of 20 in order to accomplish this task?

A. 50

B. 63

C. 35

D. 70

E. 76

26. If *m* and *n* are integers and $\sqrt{mn} = 10$, which of the following **CANNOT** be a value of $(m+n)$?

A. 25

B. 29

C. 50

D. 52

E. 101

27. The product of *x* and *y* is a non-zero constant. If the value of *x* is increased by 50%, then by what percentage must the value of *y* be decreased?

A. $66\frac{2}{3}\%$

B. 50%

C. 40%

D. $33\frac{1}{3}\%$

E. 25%

28. **(Tie Break 4)** Write $\cos 5x$ in terms of $\cos x$ only

A. $16\cos^5 x - 32\cos^3 x - 11\cos x$

B. $16\cos^5 x - 20\cos^3 x - 11\cos x$

C. $16\cos^5 x - 32\cos^3 x + 5\cos x$

D. $16\cos^5 x - 20\cos^3 x + 5\cos x$

E. None of these

29. An automobile windshield wiper 15 inches long rotates through an angle of 75°. If the rubber part of the blade only covers the last 12 inches of the wiper, find the area of this flat windshield cleaned by the windshield wiper.

A. $30\pi\ in^2$

B. $45\pi\ in^2$

C. $60\pi\ in^2$

D. $75\pi\ in^2$

E. $90\pi\ in^2$

30. PQ is 3/5 of PR. The area of ΔSQR is 15 cm^2. What is the area of rectangle PRST?

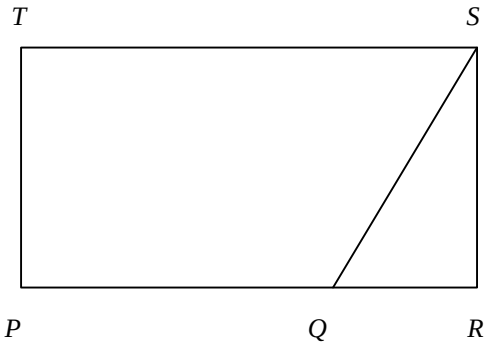
A. 50 cm^2

B. 60 cm^2

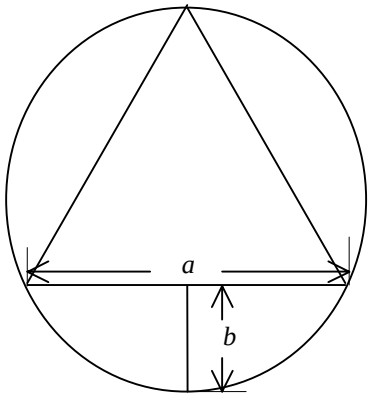
C. 65 cm^2

D. 70 cm^2

E. 75 cm^2



31. In the picture below is an equilateral triangle inscribed in a circle. *b* is the greatest distance between the arc and chord. Given that the radius of the circle is 1 what is the product of *a* and *b*?



A. $\sqrt{3}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{\sqrt{3}}{4}$

D. 1

E. None of the above

32. Reversing the digits of John's age gives his father's age. The difference in their ages is 27 years. If the sum of the digits in each age is 5, what is the sum of their ages?

A. 66

B. 55

C. 44

D. 77

E. 50

33. A circle passes though the points $(0, 0)$ and $(2, 4)$. Its center is on the line $2x - y = 5$. Find the equation of the circle.

A. $x^2 + y^2 - 4x - 3y = 0$

B. $x^2 + y^2 - 6x - 2y = 0$

C. $x^2 + y^2 - 2x - 4y = 0$

D. $x^2 + y^2 + 2x - 6y = 0$

E. $x^2 + y^2 + 4x - 7y = 0$

34. Find an equation whose roots are the negatives of the roots of the equation $x^3 - x^2 - 7x + 15 = 0$

A. $-x^3 + x^2 + 7x - 15 = 0$

B. $-x^3 - x^2 + 7x - 15 = 0$

C. $x^3 + x^2 - 7x - 15 = 0$

D. $x^3 - x^2 - 7x + 15 = 0$

E. None of these

35. $2\cos^2 x(\cos 2x - 1) - \cos^2 2x = ?$

A. $\cos^2 x + \sin x$

B. $\cos x + \sin^2 x$

C. 1

D. -1

E. None of these

36. Write the equation $r = \dfrac{3}{\sin\theta - \cos\theta}$ in rectangular form.

A. $9 = x + y$

B. $y = x + 3$

C. $x^2 + y^2 = \dfrac{3}{y - x}$

D. $y = x^2 - 3$

E. None of these

37. **(Tie Break 5)** Let A_1 be the area of a circle with a radius r, A_2 the area of the square inscribed in the circle, A_3 the area of the circle inscribed in the square, A_4 the area of the square inscribed in the previous circle, A_5 the area of the circle inscribed in the previous square, … , and so on. Find the sum of the areas of all these circles and squares.

A. $4\pi r^2 + 4r^2$

B. $2\pi r^2 + 2r^2$

C. $2\pi r^2 + 4r^2$

D. $4\pi r^2 + 2r^2$

E. Infinity

38. After finding the average of 35 scores, a student carelessly includes the average with the 35 scores and finds the average of the 36 numbers. The ratio of this average to the true average is

A. 1:1

B. 35:36

C. 36:35

D. 6:5

E. None of these

39. $\cos t = 2\tan t$. Find the values of $\sin t$.

A. $-1 \pm \sqrt{2}$

B. $-1 - \sqrt{2}$

C. $\sqrt{2} - 1$

D. $1/2$

E. None of these

40. If $f(x) = ax^2 + bx + c$, how must a and b be related so that the graph of $f(x-3)$ will be symmetric about the *y*-axis?

A. $a = b$

B. $b = 0$, a is any real number

C. $b = 3a$

D. $b = 6a$

E. $a = b/9$

41. The mean of 7 natural numbers is 10, and the unique mode is 27. Which of the following is the value of the median?

A. 3.5

B. 4

C. 5

D. 6

E. Insufficient information given

42. Let *f* and *g* be functions with continuous derivatives. $f(x)$ has a local maximum at $(0, 2)$ and $g(x)$ satisfies $g(0) = 0$ and $g'(0) = 1$. Find $\lim_{x \to 0} \dfrac{[g(x)]^2}{x^2[f(x)]^2}$

A. 0

B. $1/4$

C. $1/2$

D. $1/\sqrt{2}$

E. The limit does not exist.

43. Let *p* be an odd whole number and let *n* be any whole number. Which of the following statements about the whole number $\left(p^2 + np\right)$ is always true?

A. It is always even

B. It is always odd

C. It is even only if *n* is even

D. It is odd only if *n* is odd

E. It is odd only if *n* is even

44. If $f(x, y) = 3x + 2y$ and $g(x, y) = x^2 - y^2$ for all real numbers *x* and *y*, then $f(g(1, 2), 3) = ?$

A. 40

B. –3

C. 9

D. 6

E. 3

45. Find the domain of $f^{-1}(x)$ if $f(x) = \frac{2x+1}{x-1}$.

A. $\{x \mid x \neq 1\}$

B. All real numbers

C. $\{x \mid x > 1\}$

D. $\{x \mid x \neq 2\}$

E. $\{x \mid x \neq 0\}$